A Classical Complex 4-Wave Foundation

of the

Cosmic-Quantum Mechanism

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Fundamental Rest Mass Quanta as
Simple Harmonic Oscillations of
the Spacetime Continuum
at Resonant Frequency and Wave Number,
Driven by Cosmic Expansion

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Abstract

A model of a fundamental ½ spin quantum as a simple harmonic oscillation of an expanding 3-space of variable inertial density and resonant frequency in an underlying 4-continuum is developed. Expansion provides a mechanical analogue of an EMF which drives the neutral quantum. Absent inertial confinement, a differential decrease in inertial density creates a discontinuity, inducing a decrease in frequency to that of the proton, with transmission of the electron. Quantum gravity arises as the derivative of the wave force with respect to the expansion tension stress, and the Planck area as the derivative of the fundamental cross-sectional scale with respect to a change in stress. An exponential Hubble rate is coupled with the differential wave force and thereby beta decay. The nature of matter and anti-matter as inductive and capacitive states, respectively, is straightforward. A quantum mechanism, with animation, modeling the above is developed with the derivation of an inertial constant, $\text{ת}(tav) = \hbar / c$. An orthogonal matrix of the wave symmetries, functions, invariants, and their couplings is examined, clearly showing the relationship of the electromagnetic and gravitational interactions.

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1 – Background and Fundamentals

Motivation

Based on a presumed unity on some ontological level, an understanding of the fundamental phenomena that inform the physical world would appear to hinge on the ability to link the classical realm of large aggregates of matter and the quantum world of individual particle interactions. It requires that we find an expression of the very large, gravity, as a quantum effect and of the very small, individual particles of mass and energy, as a cosmic effect. Currently, the first of these attempts focuses on the Planck scale, held to comprise the fundamental, discrete units of space, time and mass, and the second, on the high energy conditions held to dominate at a point or locus of cosmic inception. We will try a fresh approach which derives the phenomenology of quantum effects, including gravity, from the wave bearing ontology of a cosmic continuum, which we will discuss briefly in qualitative terms.

We would expect to find the unification of the large and the small amenable to mathematical expression in an equation uniting the basic invariants of each. We might expect to find a solution to the following, in which the familiar energy-mass equation of relativity and energy-frequency equation of quantum mechanics are joined. Thus, the inherent energy of a particle, *E*, equal to the mass, *m*, times the square of the speed of light, c^2, is equated qualitatively with Planck's quantum of action, $\hbar$, times the angular frequency of a particle, ω. To elucidate this procedure, we look for a constant that couples the two expressions and find a candidate in the inertial constant, ת (tav), which will be subsequently derived, and which we introduce provisionally now as

$$\text{ת} = \frac{\hbar}{c} \,. \tag{1.1}$$

There is nothing new in this coupling of h-bar and the speed of light, but it has, to this writers knowledge, never been identified, by any name or symbol, as an invariant of significance in its own right. We will subsequently see the wisdom in doing this.

The presence of ω is an indication that a quantum particle is some manner of oscillation, and since it appears to have a discrete value over some interval of time, we will assume that it is an instance of simple harmonic motion, i.e. that there are no harmonic overtones. It is further presumed that in the context of such periodic phenomena, as in the case of a traveling wave, the following equation for the wave velocity applies, in which κ is the angular wave number, hereinafter simply referred to as "wave number", and $\partial\theta$ is the change in the wave phase commensurate with a change in time, for ω, or in space, for κ, along the length of the wave propagation;

$$c = \frac{\omega}{\kappa} = \frac{\partial\theta}{\partial t}\frac{\partial x}{\partial\theta} = \frac{\partial x}{\partial t} \tag{1.2}$$

This applies even if we envision the oscillation to be more or less fixed at a locus in space. That is, if there is an actual motion on some scale associated with the oscillation and not just a periodic phenomena emanating from a <u>point</u> in space, if it is an actual standing wave of some sort, then it will have a wave number just as would a traveling wave.

Assuming the following equivalence,

$$E = mc^2 = \hbar\omega \tag{1.3}$$

yields the rest mass as the product of the inertial constant and the wave number,

$$m = \frac{1}{c^2}\hbar\omega = \text{ת}\kappa\ . \tag{1.4}$$

This says that, on a quantum level, mass is a measure of the wave number of an oscillation.

We will next assume that the wave nature of an individual particle is an indication of its basic structure and not simply a statistical artifact, i.e. that such particles are not points, but have some inherent size as exhibited by a wave amplitude and length; that the Compton wave length over 2π, indicated by lambda-bar with a subscript C, $\bar{\lambda}_C$, as found in the "The 2002 CODATA Recommended Values of the Fundamental Physical Constants, Web Version 4.0", by the National Institute of Standards and Technology is an expression of such, so that

$$\kappa_C = \frac{1}{\bar{\lambda}_C}\ . \tag{1.5}$$

A review of the CODATA values for particle mass and $\bar{\lambda}_C$ for the neutron, proton, electron, muon, and tau, confirm (1.1) and (1.4) and show that in all cases

$$\frac{m_{particle}}{\kappa_{C,particle}} = m_{particle}\bar{\lambda}_{C,particle} = \frac{\hbar}{c} = \text{ת} = 3.51767\ldots x10^{-43}\,kg\cdot m\ . \tag{1.6}$$

This is indicative of the fact that particle mass is a measure of the wave number of a fundamental particle oscillation. It is the modulus, $\hat{\mathbf{r}}$, of some manner of quantum complex wave, which we will investigate, and we might state

$$\bar{\lambda}_{C,particle} = \hat{\mathbf{r}}_{particle}\ . \tag{1.7}$$

Assuming a physical meaning of this relationship as to the nature of an actual oscillation indicates that three dimensional space is a wave bearing continuum of inertial-elastic properties, from which the mass, m_0, of a <u>fundamental</u> oscillation at resonant frequency, ω_0, is derived, where

$$\frac{\text{ת}}{c} = \frac{m_0}{\omega_0} \tag{1.8}$$

is an invariant of the system. That is, a particle is a sustained, confined oscillation of a local volume strain of the medium, which derives its mass from the inertial properties of that medium as indicated by its oscillatory wave number.

We will return to this derivation in a moment, but first we should provide some motivation from the large scale world for pursuing this line of reasoning. In classical Newtonian dynamics, the strength or magnitude of gravitational attractive force, F_g, is directly proportional to the product of the mass, M_a, of two interacting bodies and inversely proportional to the square of the distance, d, separating their centers of mass times an empirically determined gravitational constant, G_N. This finds mathematical expression in Newton's law of universal gravitational attraction generally stated as

$$F_g = \frac{M_1 M_2}{d^2} G_N \tag{1.9}$$

We would like to find some natural derivation of G_N , mathematical or geometrical, independent of empirical determination. Based on the observation that the greatest example of gravitational force, as found in an inertial sink, i.e. a black hole, appears to be a neutron star that has exceeded a certain critical mass, and that the neutron is the more massive of the two atomic nucleons, the other being the proton, we will forward the provisional postulate that this particle serves a principle role in the operation of gravity on the quantum scale. We will assume for a minute that (1.9) is operational at that scale, so that the maximum quantum gravitational force between two nucleon would be anticipated between two neutron of mass m_n, in contact, which we will take to mean at a distance of separation of their centers of oscillation of twice their $ƛ_C$. In this regards it is further provisionally assumed that, on a quantum level, the force attributed to gravity is a manifestation of the centripetal force associated with the spin angular momentum of the particle, $\hbar$, and its oscillatory transverse wave force, so that it is operating at a distance $\hat{\mathbf{r}}$ (using spherical co-ordinates), upon the "surface" of the adjacent oscillation. Thus (1.9) becomes

$$F_{(n \cdot n)g} = \frac{m_{n1} m_{n2}}{ƛ_{C,n}^2} G_N = \frac{m_{n1} m_{n2}}{\left|\hat{\mathbf{r}}_n\right|^2} G_N \tag{1.10}$$

where the n in the suffixes is for *neutron*. Referring to the CODATA source again, for all values on the right, we find that this evaluates to

$$F_{(n \cdot n)g} = 4.24425\ldots x10^{-33}\, N \cong \frac{1}{6\sqrt{3}} ƛ_{C,n}^2 \,. \tag{1.11}$$

within a factor of

$$\frac{\frac{1}{6\sqrt{3}} ƛ_{C,n}^2}{F_{(n \cdot n)g}} - 1 = 0.000014648\ldots \tag{1.12}$$

of the CODATA value of $ƛ_{C,n}^2$ divided by $6\sqrt{3}$, which is within the relative standard uncertainty for G_N at 0.00015. A very close approximation to this factor will crop up again with respect to other constants, indicating that any measurement process that includes G_N somewhere in its mix could involve this discrepancy. The numerical co-efficient will be derived in a moment. The very close agreement of this number with the magnitude of the particle's Compton wavelength, is not found if the same procedure is used for the proton, electron, muon, and tau. This suggests a fundamental tie-in with the geometries, i.e. the curvature of the neutron. It will be noted that while the magnitude of $F_{(n \cdot n)g}$ (times $6\sqrt{3}$) is equal to $ƛ_{C,n}^2$, it evaluates in SI units of force or mdt^{-2} as Newton, whereas $ƛ_{C,n}^2$ is in units of length squared, d^2, or area. This hints at the derivative nature of $F_{(n \cdot n)g}$, which judging by the units involved is a change in force, dF, per change in stress, dT, where stress is defined as a force operating on a cross-sectional or surface area of a volume, the sine of the angle of incidence of said force to the plane of said area varying anywhere from 0, in which the stress is a pure shear stress, to 1, in which the stress is a pure tension stress, with any combination possible between these two extremes. Thus, with a little basic calculus, we have the scalar differential form

$$\frac{F}{A} = T \therefore F = AT,\ dF = AdT \tag{1.13}$$

The derivative form, with the dimensional units expressed in SI terms, is

$$\frac{F_{(n\cdot n)g}}{\text{1 unit of tension}} = \frac{dF}{dT} = \frac{d\left(kg\cdot m/s^2\right)}{d\left(\dfrac{kg\cdot m/s^2}{m^2}\right)} = \frac{\alpha\left(kg\cdot m/s^2\right)}{1\left(\dfrac{kg\cdot m/s^2}{m^2}\right)} = \alpha m^2 \tag{1.14}$$

α in this case is a number, corresponding to the very small number in (1.11) and is, in mathematical terms, the tangent or slope of the curve, $F = AT$ at the point (T,F), and if A is constant, it is the (linear) curve. It bears acknowledging what is generally implicit, that while the derivative as stated in the second term of (1.14) is taken at the limit, where both dF and dT are exceedingly small, the quantity α is the number of units of force, in this case Newton, per one unit of Pascal or Newton per square meter. Thus the derivative is always normalized or reduced to a unit value of the independent variable as the tangent in the following example is normalized as

$$\mathbf{tan}\frac{\pi}{3} = \frac{\mathbf{sin}\frac{\pi}{3}}{\mathbf{cos}\frac{\pi}{3}} = \frac{\mathbf{.}86602\mathbf{...}}{\mathbf{.}5} = \frac{\sqrt{3}}{1} \tag{1.15}$$

$F_{(n\cdot n)g}$ then can be seen as a differential change in a stress force, and as a quantum of gravitational force, G_q as

$$G_q = F_{(n\cdot n)g} = \alpha m^2\left(1 pascal\right) = \alpha Newton \tag{1.16}$$

Newton's equation can then be stated in a quantum form as the direct product of the number of quanta (primarily nucleon) in two bodies of mass and inversely to the square of the distance separating them in quantum units times the quantum of gravity as

$$F_g = \frac{n_{M1}n_{M2}}{n_{\hat{\mathbf{r}}}^2}G_q \tag{1.17}$$

In the full development of this line of reasoning, it will be shown that the proton is simply the neutron which has transmitted a portion of its energy in a wave form that we know as the electron, with a similar analysis for the anti-proton and positron. The number of quanta then is the number of fundamental oscillators or nucleon in each body of mass.

From (1.10) we can see that

$$G_N = \frac{\left|\hat{\mathbf{r}}_n\right|^2}{m_n^2}G_q = \frac{G_q}{\lambda_0^2} \tag{1.18}$$

where lambda, (not to be confused with the Compton wavelength) is the linear inertial density of a wave bearing medium or

$$\lambda_0 = \frac{m_n}{\left|\hat{\mathbf{r}}_n\right|} \tag{1.19}$$

With substitution of the following definitions concerning aggregate mass, M_a and distance, d,

$$n_{Ma} \equiv \frac{M_a}{m_n} \tag{1.20}$$

$$n_{\hat{\mathbf{r}}} \equiv \frac{d}{\left|\hat{\mathbf{r}}_n\right|}, \tag{1.21}$$

(1.17) becomes (1.9),

$$F_g = \frac{M_1 M_2}{d^2}\left(\frac{\left|\hat{\mathbf{r}}_n\right|^2}{m_n^2}\right) G_q = \frac{M_1 M_2}{d^2} G_N \,. \tag{1.22}$$

Therefore, the possibility of deriving Newton's equation from first principles exists, if we can find some quantum mechanism responsible for the phenomenological fact that

$$G_q = \frac{1}{6\sqrt{3}} \lambda\!\!\!^{-}{}_{C,n}^{2} \,. \tag{1.23}$$

Wave Bearing Continuum

Above we stated that the oscillatory nature of quantum particles indicates that three dimensional space is a wave bearing continuum of inertial-elastic properties, and should next provide some justification for that statement. Regardless of whether space, absent any oscillation, is defined as a void or, along with time, as part of a kinematic stage or protean backdrop of a four dimensional spacetime as in general relativity, it is generally recognized as permitting the translation of electromagnetic oscillations. While these oscillations are variously modeled as photons, radiation or rays, energy, messenger particles, and self propagating electromagnetic or EM waves, they are all viewed as traveling through space, hence in some manner space is allowing or permitting the penetration of such oscillations; therefore, it transfers stress, and we might surmise, strain. In terms of the EM wave model, they are recognized as transverse waves in which the electric fields, *E*, vary sinusoidally in phase with, hence at the same frequency as and orthogonally to the magnetic fields, *B*, clockwise when viewed from the direction of wave travel. Thus the cross product of E into B gives

$$E \times B \Rightarrow \text{direction of wave travel} \qquad (1.24)$$

Various properties of EM wave propagation are shown here.

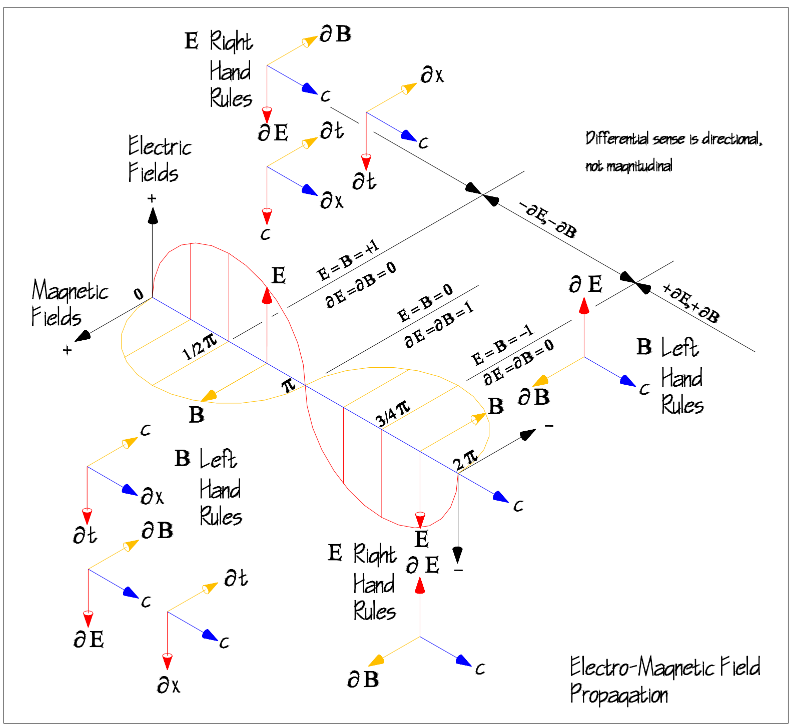


Figure 1 - Electro Magnetic Wave

It is established that for an induced electric field, using Faraday's law of induction, the change in the electric field over space is related to the change in the magnetic field over time by the scalar equation

$$\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t} = \left[\frac{-i\partial B}{i\partial t}\right]. \qquad (1.25)$$

Similarly, for an induced magnetic field, using Maxwell's law of induction, the change in the magnetic field over space is related to the change in the electric field over time by

$$\frac{\partial B}{\partial x} = -\mu_0 \varepsilon_0 \frac{\partial E}{\partial t} = \left[\frac{1}{c^2} \frac{-i\partial E}{i\partial t} \right] \tag{1.26}$$

We can shed some light on this condition and on the nature of the fundamental ½ spin particles by using the orthogonal sense of the bracketed terms to develop a vector form of these equations. While vector division is not normally defined, if we stipulate an orthogonal condition, as in the above wave, we can define vector division between orthogonal vectors, which by virtue of a ½ π rotation indicated by $\pm i$ can become parallel or anti-parallel vectors. In fact we will find that the operation of the cross product is equivalent to division involving a vector with imaginary sense in either the dividend or the divisor. This is done not to confer any notational advantage, but to indicate the underlying rotational symmetry of the system. Thus, for the right hand rule, we can state

$$\mathbf{a}(i \div)\mathbf{b} = \mathbf{b}(\div i)\mathbf{a} \equiv \mathbf{a}(\times)\mathbf{b}\,. \tag{1.27}$$

The right hand rule reverse, which is equivalent to a left hand rule is

$$\mathbf{a}(-i \div)\mathbf{b} = \mathbf{b}(\div - i)\mathbf{a} \equiv -\mathbf{a}(\times)\mathbf{b} = \mathbf{b}(\times)\mathbf{a}\,. \tag{1.28}$$

indicating that the right hand rule is equal to the left hand rule reverse.

With some rearrangement of (1.25) and (1.26) we have

$$i\partial E = \frac{\partial x}{i\partial t}\partial B = c\partial B \text{ and} \tag{1.29}$$

$$i\partial B = \left(\frac{i\partial t}{\partial x}\right)^2 \frac{\partial x}{i\partial t}\partial E = \frac{i\partial t}{\partial x}\partial E = \frac{1}{c}\partial E\,. \tag{1.30}$$

We might write this in vector terms as follows shortly, with the understanding that the i represents a counterclockwise rotation of ∂E in the plane of E-B, when viewed from the direction of wave travel, and c is a scaling or normalizing factor equal in magnitude to the speed of wave travel. In other words, if the unit of distance x was equal to the unit of time t, c would equal 1. Thus, given the right hand rule, from

$$\partial x \equiv ic\partial t \tag{1.31}$$

we have

$$\frac{\partial \mathbf{x}}{i\partial \mathbf{t}} \equiv \partial \mathbf{t} \times_R \partial \mathbf{x} = \frac{i\partial \mathbf{x}}{-\partial \mathbf{t}} \equiv \partial \mathbf{x} \times_R -\partial \mathbf{t} = \mathbf{c} \tag{1.32}$$

indicating the inherent orthogonality between **x** and **t**. **t** can be modeled as a vector anywhere in the plane, t, orthogonal to **x**, so that i**t** represents a rotation of **t** into **x** or a collapse of the t plane onto **x**. This appears to indicate that time and space are commutative, in the sense that they are interchangeable if we can think of time as constituting an orthogonal plane about a given spatial dimension, x. This extends to the left hand rule as well as

$$\frac{-i\partial \mathbf{x}}{\partial \mathbf{t}} \equiv \partial \mathbf{x} \times_L \partial \mathbf{t} = \frac{\partial \mathbf{x}}{-i(-\partial \mathbf{t})} \equiv -\partial \mathbf{t} \times_L \partial \mathbf{x} = \mathbf{c} \tag{1.33}$$

The direction of **c** is therefore determined by the direction of $\partial \mathbf{x}$, which is logical, and not by $\partial \mathbf{t}$ which is orthogonal to all of 3-space. The inversion symmetries are

$$\frac{-\partial\mathbf{x}}{i\partial\mathbf{t}} \equiv \partial\mathbf{t}\times_R -\partial\mathbf{x} = \frac{i(-\partial\mathbf{x})}{-\partial\mathbf{t}} \equiv -\partial\mathbf{x}\times_R -\partial\mathbf{t} = -\mathbf{c} \text{ and} \tag{1.34}$$

$$\frac{-\partial\mathbf{x}}{-i(-\partial\mathbf{t})} \equiv -\partial\mathbf{t}\times_L -\partial\mathbf{x} = \frac{-i(-\partial\mathbf{x})}{\partial\mathbf{t}} \equiv -\partial\mathbf{x}\times_L \partial\mathbf{t} = -\mathbf{c} \tag{1.35}$$

In (1.32) to (1.35), we have a fixed orthogonal relationship between $\partial\mathbf{t}$, $\partial\mathbf{x}$, and **c**, which we might call a native right hand relationship as given by the first crossed term in (1.32), even though we have a symmetry of right and left hand cross products. Note that there is no $\partial\mathbf{t}\times_L \partial\mathbf{x}$. With a native left hand relationship, $\partial\mathbf{t}$ and $\partial\mathbf{x}$ are transposed, as found in the Left Hand Rules section with the magnetic fields in the EM wave diagram above, and we have

$$\partial x \equiv -ic\partial t \tag{1.36}$$

$$\frac{\partial\mathbf{x}}{-i\partial\mathbf{t}} \equiv \partial\mathbf{t}\times_L \partial\mathbf{x} = \frac{-i\partial\mathbf{x}}{-\partial\mathbf{t}} \equiv \partial\mathbf{x}\times_L -\partial\mathbf{t} = \mathbf{c} \tag{1.37}$$

$$\frac{i\partial\mathbf{x}}{\partial\mathbf{t}} \equiv \partial\mathbf{x}\times_R \partial\mathbf{t} = \frac{\partial\mathbf{x}}{i(-\partial\mathbf{t})} \equiv -\partial\mathbf{t}\times_R \partial\mathbf{x} = \mathbf{c} \tag{1.38}$$

$$\frac{-\partial\mathbf{x}}{-i\partial\mathbf{t}} \equiv \partial\mathbf{t}\times_L -\partial\mathbf{x} = \frac{-i(-\partial\mathbf{x})}{-\partial\mathbf{t}} \equiv -\partial\mathbf{x}\times_L -\partial\mathbf{t} = -\mathbf{c} \tag{1.39}$$

$$\frac{-\partial\mathbf{x}}{i(-\partial\mathbf{t})} \equiv -\partial\mathbf{t}\times_R -\partial\mathbf{x} = \frac{i(-\partial\mathbf{x})}{\partial\mathbf{t}} \equiv -\partial\mathbf{x}\times_R \partial\mathbf{t} = -\mathbf{c} \tag{1.40}$$

In both the left and right hand native relationships, a positive **c** results from a rotation of one vector parallel into the other, while the negative **c** results from a rotation of that vector anti-parallel into the other. Notice that an *i* in both the dividend and the divisor is not defined as a cross product, though it might be depending on the context. The sense of the imaginary designation, then indicates by convention whether the crossing is to the left, clockwise, or to the right, counterclockwise, and its position shows it as an operator crossing the following vector into the other component of the quotient. Thus we have the following identity that is tacit in (1.31) through (1.40),

$$\mathbf{c} = \frac{\partial\mathbf{x}}{i\partial\mathbf{t}} \equiv \partial\mathbf{t}\times_R \partial\mathbf{x} \equiv \frac{i\partial\mathbf{t}}{\partial\mathbf{x}} = \frac{1}{\mathbf{c}} \tag{1.41}$$

With this in mind, returning to (1.29) we can state with reference to the diagram,

$$\frac{i\partial\mathbf{E}}{\partial\mathbf{B}} \equiv \mathbf{c} \equiv \frac{\partial\mathbf{x}}{i\partial\mathbf{t}} = \frac{i\partial\mathbf{x}}{\partial\mathbf{t}}. \tag{1.42}$$

for the transverse wave speed and direction as well as the longitudinal phase speed and direction. A similar identity holds for (1.30), if we multiply through by -1, as

$$\frac{-i\partial\mathbf{B}}{\partial\mathbf{E}} \equiv \frac{1}{\mathbf{c}} \equiv \frac{-i\partial\mathbf{t}}{\partial\mathbf{x}} \tag{1.43}$$

This follows the left hand rule, as indicated by the fact that the translational sense of **c** in (1.43) is unaffected by the rotational sense change as in comparing (1.37) and (1.32). In right hand form this is

$$\frac{\partial\mathbf{B}}{i\partial\mathbf{E}} \equiv \frac{1}{\mathbf{c}} \equiv \frac{\partial\mathbf{t}}{i\partial\mathbf{x}} = \frac{i\partial\mathbf{t}}{\partial\mathbf{x}} \tag{1.44}$$

which is simply the inverse of (1.42). In more familiar form, using the right hand rule in the cross, both (1.42) and (1.44) are

$$\partial\mathbf{E}\times\partial\mathbf{B}=\mathbf{c}=\partial\mathbf{t}\times\partial\mathbf{x}=\partial\mathbf{x}\times\partial\mathbf{t} \tag{1.45}$$

and we can see that by virtue of (1.32) and (1.38) that crossing involving time is commutative.

Clearly in Maxwell the product of the permeability, μ_0, and the permittivity, ε_0, constants as c^2 inverts the time and space differentials and relates a change in the electric fields to a change in space and a change in the magnetic fields to a change in time, precisely as in Faraday. It further shows, in keeping with (1.32), that

$$c^2=\left(\frac{\partial\mathbf{x}}{i\partial\mathbf{t}}\right)\cdot\left(\frac{\partial\mathbf{x}}{i\partial\mathbf{t}}\right)=\left(\frac{\partial\mathbf{x}}{i\partial\mathbf{t}}\right)\cdot\left(\frac{i\partial\mathbf{x}}{\partial\mathbf{t}}\right) \tag{1.46}$$

as a scalar, where the second term is a parallel dot product and the third is an anti-parallel dot product, this latter case only if $\partial\mathbf{t}$ and $\partial\mathbf{x}$ represent the same vectors in each quotient. If they do not as in the crossing diagrams representing the transverse wave speed in the EM wave diagram above, we would have

$$-\mathbf{c}^2=\left(\frac{\partial\mathbf{x}}{i\partial\mathbf{t}}\right)\times\left(\frac{\partial\mathbf{x}}{i\partial\mathbf{t}}\right)\neq\left(\frac{\partial\mathbf{x}}{i\partial\mathbf{t}}\right)\times\left(\frac{i\partial\mathbf{x}}{\partial\mathbf{t}}\right)=\mathbf{c}^2 \tag{1.47}$$

The use of identities indicates that as normalized vectors, where the time and distance scale are equal, $\partial\mathbf{E}\leftrightarrow\partial\mathbf{t}$, are interchangeable as are $\partial\mathbf{B}\leftrightarrow\partial\mathbf{x}$. Thus

$$\partial\mathbf{E}\times\partial\mathbf{B}=\partial\mathbf{E}\times\partial\mathbf{x}=\mathbf{c}\text{ and} \tag{1.48}$$

$$\partial\mathbf{t}\times\partial\mathbf{x}=\partial\mathbf{t}\times\partial\mathbf{B}=\mathbf{c}\,. \tag{1.49}$$

Also equivalent, applying the RHR to the magnetic field vectos are $\partial\mathbf{E}\leftrightarrow\partial\mathbf{x}$ and $\partial\mathbf{B}\leftrightarrow\partial\mathbf{t}$ so that

$$\partial\mathbf{E}\times\partial\mathbf{B}=\partial\mathbf{E}\times\partial\mathbf{t}=\mathbf{c}\text{ and} \tag{1.50}$$

$$\partial\mathbf{x}\times\partial\mathbf{t}=\partial\mathbf{x}\times\partial\mathbf{B}=\mathbf{c}\,. \tag{1.51}$$

This would be so much nonsense, with every vector shown as an orthogonal identity with every other, were it not for the fact that what is being elucidated is the simple orthogonal geometry of spacetime itself, and particularly of time with respect to all of 3-space. If we rotate the EM wave in the diagram ¼ π clockwise about the direction of travel, **c**, the amplitudes of E and B can be viewed as the real and imaginary amplitudes corresponding to a complex modulus, $R=\sqrt{2}E=\sqrt{2}B$, which represents the amplitude of a sinusoidal traveling wave with a transverse displacement in the *R*-*c* plane. The wave, then, is analogous to a traveling wave on an ideal stretched string, but instead of a displacement of a medium of finite and much smaller cross-section relative to the amplitude, we have a sinusoidal strain in a medium in which continuity extends the cross-section indefinitely, far exceeding the magnitude of the amplitude. The handedness of the wave would appear to be a function of the matter particles and fields that interact with the wave and not of the wave itself.

This condition is shown in the above diagram of the EM wave. The E, right hand rules group of three cross products at the top of the diagram shows the differential vectors which apply at the end of the positive electric field vector shown in the second quadrant phase, between ½ π and π. If we had used a vector between 0 and ½ π, these three would be rotated one π turn about the blue vectors, **c** and $\partial\mathbf{x}$. The differentials reverse direction at the antinodes at ½ π and 1½ π, instantaneously vanishing, and reach their maximum, indicated by 1, at the nodes. The cross product at the top and the one at four o'clock from it are identical, but the situation would be unchanged if we exchanged the $\partial\mathbf{t}$ and $\partial\mathbf{x}$ in the second one in keeping with (1.32) and (1.38). The third product, directly beneath the first, has been rotated so that **c** is shown as the transverse wave velocity vector. Thus we have two directions of wave speed, transverse for the physical motion, and longitudinal for the wave phase as shown by the blue **c**s. An analogous situation is shown for the magnetic fields at <u>B, left hand rules</u>.

Recognizing that a four minute/mile and ¼ mile/minute represents the same velocity,

$$\mathbf{c} = \frac{1}{\mathbf{c}} \tag{1.52}$$

$$\therefore \mathbf{c}^2 = \mathbf{c}\times\mathbf{c} = \mathbf{1} = \mathbf{c}\text{, a vector or} \tag{1.53}$$

$$\mathbf{c}^2 = \mathbf{c}\cdot\mathbf{c} = 1 = c\text{, a scalar} \tag{1.54}$$

depending on whether the squaring indicates an orthogonal condition, i.e. a cross product, as when taking the product of two sides of a square, or a dot product, in which case the product is, conventionally, no longer a vector. We can make it a vector once more by applying the gradient, so that

$$\nabla c = \mathbf{c} \tag{1.55}$$

Actually, implicit in (1.53) is that the cross product is as in (1.47) and therefore we have,

$$\mathbf{c}^2 = i\,\mathbf{c}\times\mathbf{c} = \mathbf{1} = k\,\mathbf{c} \tag{1.56}$$

where the k indicates that the product, **c**, is orthogonal to both of the other two **c**'s. Alternatively we can express this as

$$i\,\mathbf{c}\times j\,\mathbf{c} = k\,\mathbf{c} \tag{1.57}$$

giving the second **c** in the cross product its own orthogonal sense, or we could use subscripts

$$\mathbf{c}_i\times\mathbf{c}_j = \mathbf{c}_k \text{ or } \mathbf{c}_1\times\mathbf{c}_2 = \mathbf{c}_3\,. \tag{1.58}$$

In the case of the dot product, then, we simply have

$$\mathbf{c}_1\cdot\mathbf{c}_1 = c \tag{1.59}$$

and the gradient becomes

$$\nabla_1\left(\mathbf{c}_1\cdot\mathbf{c}_1\right) = \mathbf{c}_1\,. \tag{1.60}$$

In the case of a normalized c, (or **c**), t is simply another instance of x_ι, where

$$x_\iota = \left(x, y, \text{ or } z\right) = \left(x_i, x_j, \text{ or } x_k\right) = \left(x_1, x_2, \text{ or } x_3\right) \tag{1.61}$$

and any two orthogonal x are interchangeable, and we can go with the established convention and call t, x_0, or

$$x_\iota = \left(t, x, y, \text{ or } z\right) = \left(x_i, x_j, x_k \text{ or } x_l\right) = \left(x_0, x_1, x_2, \text{ or } x_3\right) \tag{1.62}$$

In the second of these scenarios, we placed the time dimension at the end of the sequence or shifted the subscripts to the left, depending on your perspective, showing that time and space really are interchangeable. A velocity or any other kinematic derivative is simply the rate of change in one dimension with respect to a change in another. In the above treatment, *i*, by itself and without the other subscripts, is considered a generic orthogonal sense or operator, independent of any formal complex notation. It is simply directed at an angle of $^{\pi}/_{2}$ with respect to the other term in a binary operation. If it is constrained in a plane, then it itself can be either + or -, conventionally counterclockwise or clockwise, as directly viewed, the mirror image or "view from behind" being reversed. (This indicates that in addition to an intrinsic degree of freedom, $+\,^{\pi}/_{2}$ or $-\,^{\pi}/_{2}$, and an extrinsic infinitely variable degree of freedom determined by whatever constrains the plane, there is an observational degree of freedom set by the view sense.) If there is no planar constraint, then the extrinsic degree becomes intrinsic, and *i* indicates any direction in a plane which is orthogonal from the original direction. Whether it is $+\,^{\pi}/_{2}$ or $-\,^{\pi}/_{2}$ is determined by observation or some other constraint, i.e. whether a right-hand or left-hand rule is applied. With this in mind, it bears noting that while $-iE$ and iB represent rotations within the E-B plane, $^{+}/_{-}\ ix$, and in some contexts, $^{+}/_{-}\ it$, represents a rotation into that plane from the x axis.

From this development we can combine (1.42) and (1.44), for the transverse wave speeds, keeping in mind (1.52), to get

$$\left(\frac{i\partial\mathbf{E}}{\partial\mathbf{B}}\right)\times\left(\frac{-i\partial\mathbf{B}}{\partial\mathbf{E}}\right)=\left(\frac{\partial\mathbf{x}}{i\partial\mathbf{t}}\right)\times\left(\frac{i\partial\mathbf{t}}{\partial\mathbf{x}}\right)=\mathbf{c}\times\frac{1}{\mathbf{c}}=\mathbf{c}^{2}=\mathbf{1}\,. \tag{1.63}$$

Thus the rotational change in the electric field as a function of a change in the magnetic field times the rotational change in the magnetic field as a function of the change in the electric field is an invariant vector, **1**. From (1.53) and from (1.54) and (1.60) there are two versions of this scenario, which we will explore in a moment.

The left hand term of (1.63) indicates that the induced changes in the electric and magnetic fields in some manner cancel over time and distance along the path of the wave's travel until, at the point of what we would recognize as a node, the value of both **E** and **B** is 0. The transverse changes in the fields, however, do not stop at that point, since the 0 point is a relative zero, a point of equilibrium, and the functions are continuous through such point, $\partial\mathbf{E}$ and $\partial\mathbf{B}$ being at a maximum with respect to $\partial\mathbf{t}$ or $\partial\mathbf{x}$.

We can imagine this as two blades of a pair of scissors that start in a position orthogonal to each other and are brought together toward a point 45° from each. As they come together, due to a common pivot point outside the edge of each blade, their open edges each become shorter, until they vanish at the end of their travel. The use of the scissors metaphor is not accidental. Though our treatment here uses vectors, the wave can also be modeled as a tensor field.

Thus the *E* and *B* fields can be modeled to represent the shear stress components and corresponding strains of a stress and a strain tensor, where **c** in the direction of travel, represents the tension components. The wave form of both fields is sinusoidal, at $^{\pi}/_{2}$

rotation from each other as indicated above. At the point of maximum shear strain and stress, corresponding with the amplitude of the wave, the transverse wave momentum instantaneously vanishes, just as the transverse, shear stresses instantaneously vanish at the node. **c** then, while invariant with respect to the speed of the wave phase, has a transverse component that oscillates and reaches a maximum of 1 as both **E** and **B** reach 0.

As we emerge through the antinodes at ½ π and 1½ π, (1.63) becomes

$$\left(\frac{-i\partial\mathbf{E}}{-\partial\mathbf{B}}\right)\times\left(\frac{+i\partial\mathbf{B}}{-\partial\mathbf{E}}\right)=\left(\frac{-i\partial\mathbf{x}}{\partial\mathbf{t}}\right)\times\left(\frac{i\partial\mathbf{t}}{\partial\mathbf{x}}\right)=\mathbf{c}^2=\mathbf{1} \tag{1.64}$$

leaving the product unchanged, although the field directions have all reversed, having multiplied both terms of each quotient by -1. This differential sense change is set by the amplitude of the wave, which is a dynamic function of other variables. Of course, (1.64) reverts to (1.63) through a cancellation of the senses, and the changes in the differential senses are always simultaneous, so that **c** remains invariant, that is under the following of the two above mentioned conditions.

If we recast (1.63) in light of (1.60), we have the following normalized gradient in the direction of wave travel, which describes the radial propagation of an electromagnetic wave from some source,

$$\nabla_x\left(\left(\frac{i\partial\mathbf{E}}{\partial\mathbf{B}}\right)\cdot\left(\frac{-i\partial\mathbf{B}}{\partial\mathbf{E}}\right)\right)=\nabla_x\left(\left(\frac{\partial\mathbf{x}}{i\partial\mathbf{t}}\right)\cdot\left(\frac{i\partial\mathbf{t}}{\partial\mathbf{x}}\right)\right)=\nabla_x c^2=\mathbf{c}^2=\mathbf{1}\,. \tag{1.65}$$

Note that the second term is algebraically self normalizing.

To arrive at a dynamic expression for the wave, we need some scaling factor that will indicate its energy and mass, i.e. its angular frequency and wave number. Using (1.2) and (1.4) we have

$$E=\sqcap\mathbf{k}\cdot\nabla_x c^2=\sqcap c\omega=\hbar\omega\,. \tag{1.66}$$

We would like now to see if there might be another application of (1.63) using the cross product. We assume, for purposes that we will later make clear, that the wave radiates from some locus, so that at that source, the directions of propagation can be resolved in terms of three co-ordinates, *x*, *y*, and *z* as shown in the following subscripts. We would expect, therefore, that (1.63) would take the form

$$\mathbf{c}_x\times\mathbf{c}_{-z}=\mathbf{c}_y \tag{1.67}$$

where we choose subscripts which best serve our long term purpose.

In keeping with the orthogonal nature of the electric and magnetic field orientation, and their cross product which results in a third orthogonal vector, (1.67) requires that <u>either</u> the electric <u>or</u> the magnetic fields and their differentials be common to both vectors $\mathbf{c}_x$ and $\mathbf{c}_{-z}$, unless the wave can be modeled as a 4-D wave. Thus we will make the field orientations (not that of the differentials) explicit with the appropriate subscripts. Since there is no evidence that magnetic monopoles exist, we will assume that at the source locus a magnetic dipole does exist. This implies that any electrical fields radiate at and from the center of the dipole perhaps as a ring of quantum charge. All we are interested in are two such fields orthogonal to each other. One such configuration that might satisfy this relationship is

$$\left(\frac{i\partial \mathbf{E}_z}{\partial \mathbf{B}_{-y}}\right)\times\left(\frac{-i\partial \mathbf{B}_{-y}}{\partial \mathbf{E}_x}\right)=\left(\frac{\partial \mathbf{x}_{-y}}{i\partial \mathbf{t}_z}\right)\times\left(\frac{i\partial \mathbf{t}_x}{\partial \mathbf{x}_{-y}}\right)=\mathbf{c}_x\times\mathbf{c}_{-z}=\mathbf{c}_y \tag{1.68}$$

We would imagine, given the dipole nature of *B* and our innate sense of the justice of symmetry, that another such simultaneous configuration exists so that

$$\left(\frac{i\partial \mathbf{E}_{-z}}{\partial \mathbf{B}_y}\right)\times\left(\frac{-i\partial \mathbf{B}_y}{\partial \mathbf{E}_{-x}}\right)=\left(\frac{\partial \mathbf{x}_y}{i\partial \mathbf{t}_{-z}}\right)\times\left(\frac{i\partial \mathbf{t}_{-x}}{\partial \mathbf{x}_y}\right)=\mathbf{c}_x\times\mathbf{c}_{-z}=\mathbf{c}_y \tag{1.69}$$

But wait! This is anything but symmetrical! Since the same right hand rule for the cross product has been used for both (1.68) and (1.69), the velocity cross products and their final cross product are the same in both cases. We will explore this further, but our intention at this point is simply to show that the same ontology which supports a classical wave mechanism found in a propagated electromagnetic wave can be modeled as the support for the quantum source mechanism, and we can model both as a spacetime continuum with inertial-elastic wave bearing properties.

Returning to (1.69), if we assume symmetry at the source, we must assume that instead of the right hand rule for the cross product, we should use the left hand rule for the field polar opposites of those in (1.68). Doing this we have

$$\left(\frac{-i\partial \mathbf{E}_{-z}}{\partial \mathbf{B}_y}\right)\times_L\left(\frac{i\partial \mathbf{B}_y}{\partial \mathbf{E}_{-x}}\right)=\left(\frac{i\partial \mathbf{x}_y}{\partial \mathbf{t}_{-z}}\right)\times_L\left(\frac{\partial \mathbf{t}_{-x}}{i\partial \mathbf{x}_y}\right)=\mathbf{c}_{-x}\times_L\mathbf{c}_z=\mathbf{c}_{-y}\,. \tag{1.70}$$

This implies a basic symmetry at the quantum source which is broken as a result of propagation of the EM wave, resulting in a preponderant phenomenology of right hand electrical, left hand magnetic, versions of such.

What this shows us about the source, by adding (1.68) and (1.70), referring to Figure 1, is:

1) the change in electrical fields as a function of a change in the magnetic field over time ∂t results in a radial tension stress along a path of potential wave propagation, $\mathbf{c}_{+x}$ - $\mathbf{c}_{-x}$ which in an equilibrium condition are balanced, preventing propagation,
2) the change in the magnetic fields as a function of a change in the electrical fields over the distance ∂x results in the velocity vectors $\mathbf{c}_{+z}$ - $\mathbf{c}_{-z}$ and a rotation of the magnetic dipole, B_{+y}- B_{-y}, making it an axial vector, in this case shown as a left hand vector, and which we will call ϕ,
3) the cross product of the velocity vectors in (1) and (2) creates the vectors $\mathbf{c}_{+y}$ - $\mathbf{c}_{-y}$ and results in a rotation about a second axial vector, this time shown as right hand by the blue arrow, S_L, which we will call θ, at E_{+z}- E_{-z}, which remains, absent any perturbations, fixed in space over time, whereas ϕ remains fixed in time, i.e. phasing, over space as it rotates about θ.

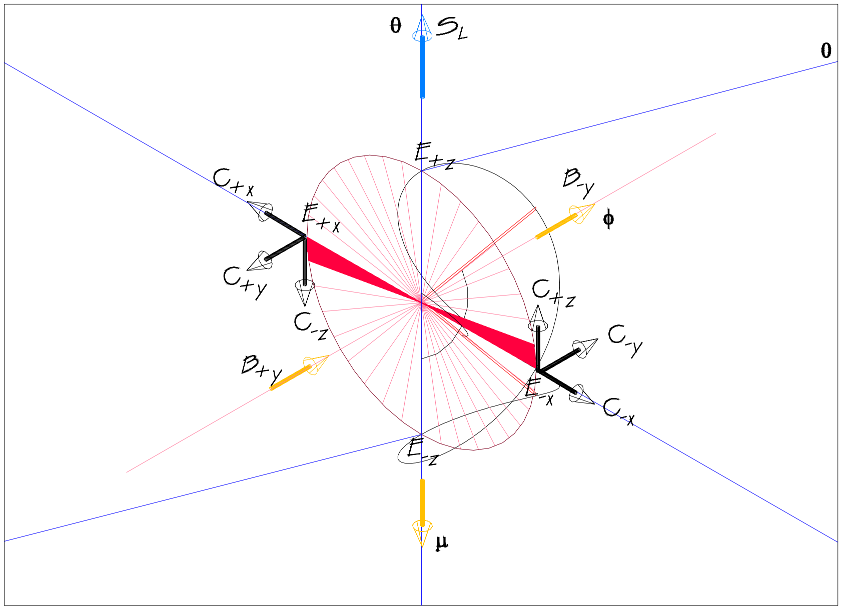


Figure 1

θ is a spin angular momentum vector while the yellow axial vector pointing to the bottom of the figure, mirrored to θ and shown as a left hand axial vector, μ, is the effective magnetic moment of the source. The angle between ϕ and the line E_{+z}-0, and the analogous one involving E_{-z} are angular strains constituting the electric fields, which oscillate in place over time, while the angle between μ and ϕ on either side of the figure is a permanent orthogonal distortion which rotates as it moves about θ. The figure 8 path that appears to be bent around the central disk on the right hand side of the figure with its center at E_{-x} is the oscillatory path of the <u>strain</u> which that point takes as ϕ rotates about θ. There are an infinite number of such, one corresponding to each point over the circumferential distance of ϕ and with each corresponding to two points on the circumferential path of θ over one 2π cycle. Thus the stress rotates about θ while the strain oscillates along the figure 8 path.

We will investigate this model in greater depth. The opposition of the spin and magnetic moments indicates that it is an electron, neutron or an anti-proton, but we will see that it in fact represents a neutron. There are obviously some steps between this form and the generation of electromagnetic radiation, in particular involving beta decay and the generation of an electron or positron. The significance now is that if we apply the same logic to (1.68) and (1.70) as we did to (1.65), we have the standing wave corollary of the energy equation found in (1.66) for a rest mass particle, in which factors of ${}^{1}\!/\!{\sqrt{2}}^{\,2}$ related to the root mean square for the maximum power of a wave and the two instances of instantaneous spin energy, $\mathbf{E}_{+y}$ and $\mathbf{E}_{-y}$, equal unity giving

$$\begin{aligned} E &= 2\left|\sqcap\mathbf{k}_{\pm y}\cdot\left(\tfrac{1}{\sqrt{2}}\mathbf{c}_{\pm x}\times_{R/L}\tfrac{1}{\sqrt{2}}\mathbf{c}_{\mp z}\right)\right| \\ &= \left|\sqcap\mathbf{k}_{+y}\cdot\left(\tfrac{1}{\sqrt{2}}\mathbf{c}_{+x}\times_{R}\tfrac{1}{\sqrt{2}}\mathbf{c}_{-z}\right)-\sqcap\mathbf{k}_{-y}\cdot\left(\tfrac{1}{\sqrt{2}}\mathbf{c}_{-x}\times_{L}\tfrac{1}{\sqrt{2}}\mathbf{c}_{+z}\right)\right|. \qquad (1.71) \\ &= mc^2=\sqcap c\omega=\sqcap\kappa^{-1}\omega^2=\hbar\omega \end{aligned}$$

As we will see, the spin angular momentum is modified by these power and other geometric factors, resulting in a coefficient of $\sqrt{3}/2$. The above figure implies that the oscillation pictured is an instance of simple harmonic motion. An animation of this motion is available. We will examine the kinematics and dynamics of such motion, using a model of standing wave motion on a one dimensional string.

Before we examine this oscillation more closely, a few words about continuity and isotropic expansion are in order. It should be stated that what we are talking about is not a process of inflation as currently generally conceived. As modeled here it is not a hyper rapid and extensive early epoch phenomena, but rather an ongoing process, that while slow from our frame of reference, is accelerating in an exponential manner. It would not be surprising if that exponential manner is in fact a complex exponential expansion and thus an indication of cyclical expansion and contraction.

Geometry and Topology.

Geometry deals with the subject of the shape of space and objects within that space, while topology is concerned primarily with the subject of continuity of a space and is intimately connected with the subject of set theory. These disciplines are vast, and a thorough elaboration far beyond the expertise and inclination of this writer, however, we will touch on a few matters that have some bearing on the subject under development, without undue concern for rigor.

At the heart of set theory and topology lies a paradox that receives little apparent attention and that is the nature of a point. In geometric analysis, it is readily understood that a point is dimensionless and therefore has no size. What it does have is location, so it is always described by its relationship to something else, never mind that such something else is itself thought of as a point or collection of points. This last item is the departure for set theory and topology which is content to state that a point is understood by the neighborhood or locus of other points around it.

If a point has no size, there is little point trying to add up all the contiguous points in a space to create a neighborhood, since there are as many points between one point and the point next to it as there are seconds in eternity. There is no manner of determining a scale from scratch, so to speak, in this madness. It is sufficient to draw a closed loop on a supposed continuum and state, “This is my neighborhood…”, and draw another loop connected to that one, and state, “this is an adjoining neighborhood”, etc.

The continuum is what we start with, and we partition it by defining a portion of it as separate from the rest of it, as we with all due seriousness define a portion of the good earth to ourselves and our heirs in perpetuity and build a house. The line separating us from our neighbor has no width, but we will both be aware when it is crossed, because it does have length. It has a dimensionless starting and ending point, and the line can be seen or at least envisioned by virtue of the fact that they and many other points along the continuum in between are separated by a whole lot of infinities of dimensionless points, that is, a whole lot of nothing.

It is easy enough to imagine that the widthless circumferences we use to describe our sets of mathematical neighborhoods can be shrunk to a point, and this is in fact the method that is used to distinguish the genus or different orders of topological spaces that make up an n-space or space of n dimensions. If any and all closed looped lines or analogous structures drawn on an n-space can be shrunk to a point, that n-space is considered a genus 0, and is held to be equivalent to a sphere. If some of those loops run the risk of being caught up on a donut hole or the hole forming a coffee cup handle or the n-space equivalent, they are a genus 1 n-space. If they can get caught on two different holes they are a genus 2 n-space. This is apparently pretty straight forward, and at the time of this writing a gentlemen from Russia was about to be awarded a pot full of money for proving that the genus 0 3-space is the simplest of such and equivalent to the genus 0 of the 2 and other numbered spaces. The problem is that this method of distinguishing a genus is based on a fallacy, since *you can never shrink a loop down to a point*. A loop has length and every one agrees that a length has dimension, in fact it is the very essence of dimension, but it can never be shrunk to a dimensionless entity, any more than, going in the other direction, any number of planes can be stacked to form a solid. They are simply different types of things.

To be sure, we can use calculus to integrate an area and arrive at a volume, but that is because of the underlying assumption of continuity on which the calculus is based. Without the basis of continuity, the functions and the derivatives which they father would fail. At whatever scale we try to reduce it to, a loop remains a loop and never will arrive at a point. Which is not to say that holes don't exist. Holes are null sets, just as each and every point to which a loop might approach as a limit is a null set, and one thing that both set theorists and geometers will agree on is that points and null sets do exists. But you, that is a dimensioned entity, will never get there, since there's no there, there to get to. The loop at the limit does not become the point, it surrounds the null set point and retains its loopness. Points, however, do have location, as we have said, within a given continuum, be it line, plane, volume, etc., and so does a hole, and that location is where a loop drawn around it, within a given continuum, is the smallest. In the sense of having location, therefore, neither hole nor point is a null set and both are defined within a continuum and have structure. It is in order to discuss that definition, the structure of a point and a null set, that we entered on this digression.

Poor Gödel and his ordered infinities! There are at least twice as many rational numbers as irrational, since any irrational number whose binary digits we may be counting, that is calling out or naming, has one rational number on either side of it at each digit along its span; so much for denumerable infinities. Infinity, at least in the sense of divisibility of a continuum, is the rule and not the exception, and counting is simply a way of drawing temporary loops on the expanse of it. It is just that any number of null sets added together do not make a continuum. And a continuum is both whole, that is one, and infinite at the same time, all the while being the home of every null set and conceivable collection thereof. Pretty simple when you think about it, or perhaps that is, if you stop trying to think about it.

Actually, it is not *quite* that simple. The reason the point can never be reached is that a spherical surface, a 2-sphere as it is generally called, approaches Euclidean flatness with increasing refinement of scale and paradoxically flatness has no scale. Even the curvature of the loop, by itself, gives no concept of scale. At some point the continued shrinking of the loop and the shrinking adjacent field of reference, our tunneling scope of observation, if they co-vary or assume a common velocity of shrinkage, takes on the quality of a state, a static condition, instead of that of change and motion.

The only way the point can be reached is for us to define it as being there. We must state, "Enough of this tedium. I order you to loose the one dimensional quality of a 1-sphere (a circle), and become a dimensionless 0-sphere (a point)!" And it happens. But something else, very strange, happens. We can never place another point right next to the first point, since we can always zoom in towards the first point at a scale that finds the second receding into the distance. We can continue to zero in on the point, for a length of time equal to that already spent on this shrinking endeavor; for a day, a week, an eternity, but we never appear to get any closer to the point. At any scale it continues to appear just the same, just a point. It will never become a disk or a hole. We can make it change into cross hairs if we like, but they will not change in appearance as we approach, always of some length, but forever of immeasurably small width and inscalability. But then the reason for establishing, for pinning down a point is not possession, but rather station and partition of an otherwise protean and irrational continuum, and this can be done to a reasonable precision and accuracy within a few orders of magnitude.

It is curvature that grants scalability. Even the most rigorous Cartesian logic assumes an arc of continuity across the vast Pythagorean expanse between the *x* and *y* co-ordinates in staking out an ordinate and abscissa. And while nature has given us the asymptote, she has also given us the tangent and Herr Riemann to show us that the two can be in fact the same thing in curved space. Lines do touch, and just as they may be made to intersect at one point in our crosshairs, they can be made as a circle and a line, and thereby, two circles, in the world of 1-spheres embedded in a universe of 2-spheres, to intersect at just one point. . .or in some cases two.

It is relatively easy to think of a 2-sphere, the surface of a 3-ball or three-dimensional volume, as two dimensional, and their denizens as one-dimensional combinations of lines and their circumscribed 2-balls. We might consider a figure eight and its enclosed 2-balls and speculate on whether it consisted of two separate disks united at a point of tangency or a single 1-sphere that somehow had one portion inverted. There is no way on close inspection to tell an up or down, in or out side of a 2-space, which includes the 2-sphere and any number of other 2-manifolds, such as the torus, and including the Klein bottle in which the sides are continuous, so there is no way of determining if the 8 is conjoined or self-crossed.

In fact, within that 2-space, or 2-manifold, which is generally speaking a space without a boundary, the idea of an in and out is meaningless. It is only from our three dimensional perspective that the notion of an in and out arises. What does have meaning is whether or not there is a boundary to the space. A 2-sphere has no boundary, since the sides, the in and out surfaces of the "skin" that we recognize as separating the interior of the 3-ball from the exterior, do not exist for the 2-denizen and there are no edges to bump into, as for example we would find in the case of a 2-ball or disk which is bounded by a circle or 1-sphere. The boundary of a space, generally speaking, is one dimension less than the space itself, so that a 1-line segment is bounded only by its two 0-endpoints, a 2-disk by a surrounding 1-circle, a 3-ball by the surrounding 2-sphere. An n-space is not bounded by the next higher dimension, though it may be closed or open to it.

It is here, perhaps that the system gets sticky. For topology, the ordering of spaces, in addition to the reference to n-dimensions and genus number and boundary condition, allows for some fuzziness by asking whether the space is closed or open. This is not the same thing as whether or not it has a hole in it or even a boundary. It has to do with whether or not, when you strip the boundary off a ball or interior, you take any of the ball with you. It is a matter of whether the n-boundary belongs to the n+1-ball. If it is a closed space apple, it means when you peel it you take some of the apple with you. If it is an open space banana, you get only the 2-peel and leave the 3-fruit intact. With fruit this is understandable enough, but with n-space manifolds it can get tricky.

This is just another example of the open or closed interval question, which gets back to the questionable ontology of a point. It really is an issue of whether the boundary is approached asymptotically or tangentially. Can you get right up next to the boundary point and leave the point without leaving the point next to it? If it is a Euclidean space and you are approaching the open boundary asymptotically, you can never get quite close enough to avoid leaving some of the ball behind with the rind, and yet Euclidean space says that, by fiat, you can. If we are dealing with a curved space continuum which is truly connected, then any cut we make will be tangential <u>at</u> some point, and the point will remain with both the ball and the boundary space, because the n-boundary of an n+1-enclosure has no dimension in the direction normal to the cut. An n-space which is a boundary is not like a peel or a skin of an n+1-fruit of any variety. It has no thickness. In general, however, this does not appear to be topologically admitted. If you take the n-space away from the n+1-enclosure, that enclosure remains intact.

We might imagine that an n-space might be closed with respect to adjacent n+1-dimensional spaces and open with respect to an n-1-dimensional boundary. Thus an open disk or 2-ball might be morphed into a spherical bowl with a small opening at one end. Any denizen of the 2-ball, turned almost 2-sphere, as he approached this opening would be whisked along the edge, but never quite get to it. The openness implies that the edge can never quite be reached. If this edge (not the opening itself) is closed, he might reach it but move along it tangentially without recognizing it as any more limitation than the interior or exterior that only we 3-denizens can see, and that limits his motion tangentially to the almost 2-sphere itself.

It gets stickier. The notion of topology says that any space of the same dimension and genus number is in some sense equivalent. Thus with a 2-ball (a flat disk) and a 2-sphere (surface of a 3-ball), though the first is with and the second is without boundary. The geometry of the space, whether it is concave or flat, doesn’t matter. However, it does not say that you can inflate a flat disk and get a 3-ball. That is adding a dimension out of compressed air. Nor does it let you remove the boundary of the aforementioned morphed disk turned almost sphere to close the opening in the jar, no matter how small it lets you make it.

And yet it states the equivalence of a coffee cup and a donut, and lets you shape one side of the latter into the concave vessel of the former. The reason is that the torus/coffee cup/donut was always the 2-dimensional boundary for the 3-dimensional volume of stuff out of which each shape was made, while the 2-ball disk was never a boundary, at least an enclosing boundary, of any 3-d thing. Perhaps physically a drawing on paper, but conceptually without any interior, it has no 3-d substance out of which to construct the 3-d interior for a 2-d surface to be the boundary of. It <u>is</u> the interior, of a 1-sphere.

Yet all is not lost. If we can shrink a closed loop (closed in terms of set theory as well as geometrically) to a point on the surface of a sphere by approaching it tangentially, we can surely enlarge it to that point's antipode and cover the sphere, and if we can cover a sphere that is already there, surely we can trace out the same contours of empty space and create one from a closed loop boundary of a closed flat disk. The question of whether or not the gap surrounding the antipode can be sealed off depends on whether or not the boundary approaching it is closed to it or open. If the 2-disk itself is closed, that is if it "owns" its boundary, and the boundary is itself closed on both sides, that is if it is truly only 1-dimensional, the gap can close, since there is no "fuzziness" of an asymptotic approach to prevent closure. The shrinking loop reaches tangency with all the tangent lines forming the plane of tangency at the antipode and in fact becomes self-tangent. We might even think of a point, instead of as null set, as a circle which is self-tangent.

We must digress further in our topological digression from the subject of the fundamental interactions with some comments about open and closed-ness. In topology and set theory[1], the concept of the real number line, in fact the whole concept of a set, is based on the use of the Dedekind cut, which stipulates that a space (including a 1-space or line) can be cut with a parenthesis, which represents an open or asymptotic approach to the cut, or a bracket, which represents a closed or tangential approach to the cut. The cut itself, unless it is a pre-existent boundary of the space under the knife, must be approached from both sides, so a scissors must be used, giving us four possible types of cuts, represented by

I.	Closed – Open	Tangential - Asymptotic	L---](---R
II.	Open – Closed	Asymptotic - Tangential	L---)[---R
III.	Open – Open	Asymptotic - Asymptotic	L---)(---R
IV.	Closed – Closed	Tangential – Tangential	L---][---R

where the L and R serves only to emphasize the left and right hand section of the line or a left and right hand set. Unfortunately, of the four possible types, set theory only allows the first three, the first two of which are held to represent the cut at a rational number, and the third of which is held to represent the cut at an irrational number. The set of all such cuts forms the real number line and it is a feature of this line, that, by definition, if we make a cut so that x is a number in L or an element of the L set, and y is a number in R or an element of the R set, where the largest number in L will be less than the smallest number in R, then x will be less than y and either L will have a largest element or R will have a smallest element. That is, either L or R will be closed, but not both, and a number z can belong to either L or R, but not both.

This no-doubt arose in such manner because the first set theorists were accountants and a penny could either belong in one account or the other, but not both, and it could not be cut. If they had been dress makers or carpenters, they would have realized that when you cut a piece of fabric or mark off a piece of lumber, the line of demarcation belongs to each side of the partitioned space, unraveled thread and sawdust notwithstanding; or

[1] An Introduction to the Elements of Mathematics, John N. Fujii, John Wiley & Sons, New York, 1961

better yet, had they been surveyors with real live property owners to remind them of the fact that, at the very least, the line did not belong exclusively to the other side.

On a fundamental level, numbers are used for counting or for measuring; counting generally conceived as being a quantum or integer affair, and while use of a tape measure can be thought of as counting the units and fractions thereof, the notion of continuity is essential to the application of linear measure. If the distance being measured exceeds the length of the tape or measuring rod, a position mark is made on the field and that same mark is held to be the end of the previous extension and the start of the next. That mark is in the last element in the set of the first measure and the first element in the set of the next.

In the real and physical world of, from this writer perspective, undeniable continuity, the world of conservation of energy and momentum and mass, only the fourth of the above cuts takes place. To be sure, there will always be questions of precisely where the cut took place, but there is never any supposition that if one side is neatly cleaved the other is out there in never-never land, perpetually waiting for the asymptotic axe to fall. One is tempted to point to Schrödinger and Heisenberg, but theirs are epistemological issues and not ontological, as indicated by EPR and quantum entanglement.

With the above in mind, we can stipulate that a 2-ball can be charted into a 2-sphere, and if a closed loop without boundary can be made to vanish, then it can certainly be made to reappear, this time with a vengeance. By squeezing the pode and the antipode of our 2-sphere together, at the center of its 3-ball, we can create a horn torus, a torus without any hole, except for the one single point of self tangency at the center. Now, that closed point can be expanded as a closed loop toward the center circle of the toric annulus in the equatorial plane and we have a donut. From here we can form a coffee cup, and now that the genii are out of the bottle, as many holes as we desire, with no lack of continuity. And now that we have created a 3-ball of genus-n out of a 2-ball which presumably we could have created out of a 1-ball, why not fold the 3-ball-n into a 4-ball-n, etc. It is all just a continuum, after all. But let us concentrate on the horn torus for a while, which, at the risk of disturbing Decartes' spirit, will herein be referred to as a monad.

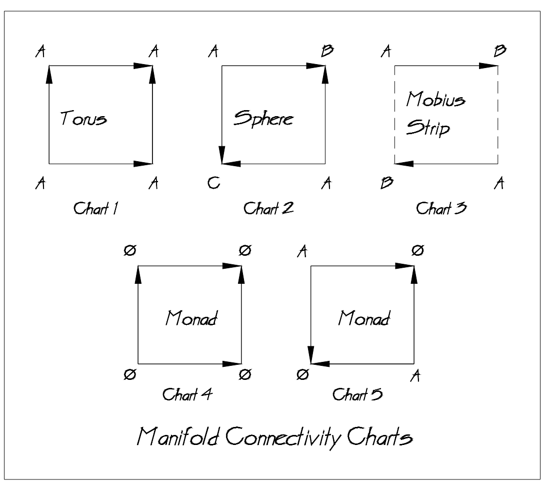
This manifold is a hybrid between a sphere and a torus, in the sense that it forms a torus for which the hole is but a single point. Since a single point has no dimension, it is effectively the null set, $\varnothing$, and as such represents the defining hole of a torus; however, as a single point maintains linear connectedness and closure between neighboring points, and here neighboring hemi-folds, this manifold is topologically equivalent to a sphere.

Of added interest is that this manifold is invertible through the center in the manner of the Möbius strip, and maintains continuity between a lower section and an upper section of the opposite lateral half. If we envision a vertical cross-section through the center of the manifold, we have a figure 8 laying on its side or the symbol for infinity, ∞, (whimsically enough, if we view the manifold from above and consider the center point to be a hole, we have the symbol for 0, while viewing it rotated 90 degrees from the exterior side, we have the symbol for 1.) We can make the ∞ by tracing two circles intersecting at a point or with one stroke in the familiar fashion, crossing at the center. If we trace the figure in this latter manner, but keep our marker to one side of the line, we will see that it in fact inverts, on the inside of one of the circles, transferring to the outside of the adjacent one as it crosses. The central crossing point thus represents a point of inflection for the space.

The central point also maintains connectedness between the upper and lower exteriors of the manifold along the tangent line, which is the central axis of a tangent sheaf or bundle of circle arcs of radius greater than that of the monad's annulus, tangent to the annular surface at the central point. Necessarily, the full circle elements of the sheaf form the set of all elements exterior to the 2-monad, whose interior we will call a 3-core to distinguish it from a 3-ball or 3-donut. The 3-core, then, can be seen as the set of all circles tangent to the central point and the central tangent axis with a radius smaller than that of the monadic annulus. Significantly, the central point also maintains 3-core diametric connectedness.

It follows that the central point is simultaneously an element of the exterior and of the interior of the 2-monad while being the central element of the 2-monad itself. In fact from the perspective of the 2-monad, it is central to all three sets. The cross section of the 2-monad forms the aforementioned figure 8, which is itself a 1-monad and it follows that the exterior, interior or 2-core, and the space itself bear the same relationship to the central point as those of the 2-monad. The two apparent disks or 2-balls of the interior are in fact continuous through the center diameter of the 1-monad. We might infer from this that the same relationship between exterior, interior and the space itself holds for all n-monads.

The following diagram of manifold connectivity is illustrative of the fact that this manifold is topologically equivalent to the other three. The charts are two dimensional Euclidean representations of the corresponding manifolds, and can be thought of as differential areas, which with integration over the manifold surface undergo the necessary transformation or stretching required to tile the surface.



Manifold Charts

i. In Chart 1, the Torus, the left and right sides join in common orientation as indicated by the direction arrows to form a tube, which then joins top to bottom to form the familiar donut shape. The repetitive corner designation, in this case A, indicates a common point in the torus manifold. We could also start by joining the top and bottom then join the sides, giving us a different orientation.

ii. In Chart 2, the Sphere, corners A can be thought of as joining at an equator, while the top and right sides and the bottom and left sides come together at the B and C poles respectively.

iii. In Chart 3, the Möbius strip, the top and bottom sides are twisted one half turn before joining to form the usual shape, while the dotted sides indicate unjoined edges.

iv. Charts 4 and 5 represent two equally valid means of composing the Monad. The first, in which the common point, $\varnothing$, is at the center, is equivalent to the Torus, while the second, in which the B and C poles are drawn together at the center, $\varnothing$, is equivalent to the Sphere and, some mental gymnastics will show, to the Möbius strip. However, instead of twisting the strip to join A and B, one common corner is joined first as when joining A to A in Chart 2, followed by a twisting through the center of the ring to joint the other two corners. The opposite sides are not joined right to left or top to bottom, however, but rather the co-terminal edges are joined as with the Sphere, top to right and bottom to left, in the case where the A's are joined first.

As this particular manifold is topologically equivalent to both a sphere and a torus and in addition the Möbius strip, and granting the closure and opening procedure of above, the Klein bottle, it can be seen as a parent of all of these in 2-space and we might conjecture in n-space and to hold the place generally reserved for the sphere. With respect to the mapping of the complex and projective plane onto the sphere, with infinity at the top pole and zero at its antipode, with the monad we have that same manifold with the infinity and zero point joined at the center of things, the unit sphere occupying the equatorial cross-sectional plane, and the negative z axis, which is the 2-space itself, forming the apple skin and the positive z axis vanishing in the core.

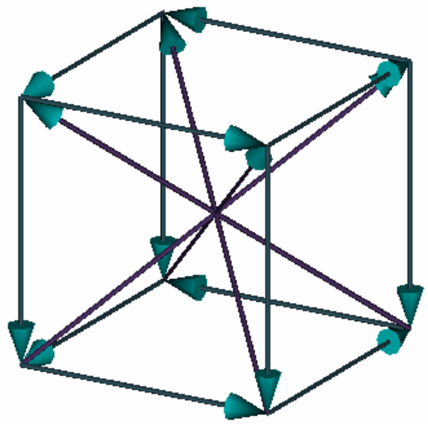
If we recognize that the imaginary sense i is simply another way of representing a half π rotation, and make of it a motive operator, we have a manifold that rotates about its central axis or horn, counter clockwise by convention, and about its annulus, upward through the horn center and downward at the periphery. That is, if we imagine the 2-monad as divided into octants, we can assign a velocity potential for each of the four octants in the top half to rotate counterclockwise viewed from above, and simultaneously outward and down on the surface of the form toward the octant below, while a simultaneous velocity potential exists for each of the lower octants to rotate in the same direction on the lower half but inward through the center and to the diagonally opposite octant on the upper half. This last motion amounts to an inversion of the 2-space surface through the center in the manner of the Möbius strip.

It remains to consider the nature of a 0-monad or monadic point and the 1-core. We can consider the 0-monad as the central point of an n-monad itself. Unlike a Euclidean point, which is generally held to be dimensionless and directionless, the 0-monad maintains an orientation or direction. This direction, which we might think of as a differential vector, is the 1-core and is tangent to the corresponding 1-monad or 2-monad of which it is an element. It is, however, for an isolated point indeterminate in its orientation, having with respect to a 3-core, 6 potential orientations or three degrees of freedom as constrained or bounded by the 8 cubic vertices which are the parameters of the 0-monad. The velocity potentials, therefore can be assigned to vectors extending from one to another of the vertices.

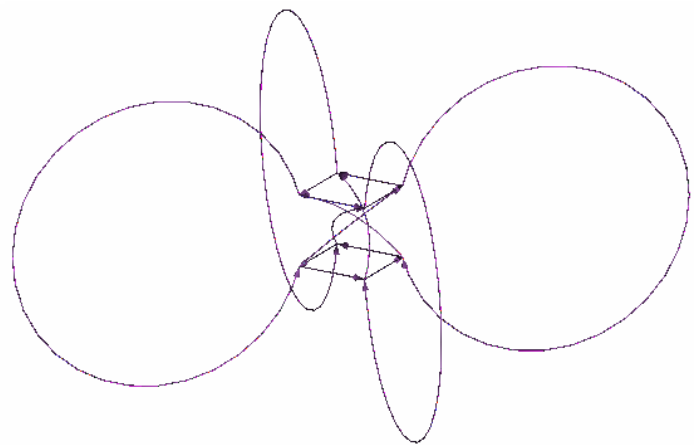
This is not a stipulation of a lattice but rather of an orthogonal potential. As an element in an n-monand, the 0-monad is closed on the 1-core and open on what would be the 2-core. That is, it is closed on the tangent line, and open on any line normal to that tangent line and point. The exception is when it is at the position of the point of central tangency, in which case it is closed on all sets, the n-monad, its n+1-cores, and its exterior, and hence is a point of continuity in all n-space. At the central point, in connection with the dynamics just indicated, it has 2 extrinsic degrees of freedom, annular rotation, either up or down, and chirality, or left or right handed, clockwise or counterclockwise rotation. Of these, only the contrast of chirality to annular rotation, hence 1 degree of freedom, is intrinsic.

If we consider the 8 vertices of the 0-monad, with regard to the various possible flow patterns or velocity potential vectors connecting them, we can see that the above scenario involving the use of *i* as a motive operator can be mapped onto the 0-monad, so that we have the following table of possible flows for a right hand, counter-clockwise, +*i* convention. The related rotations indicate a rotation of the unit vector as seen from the positive axis normal to the rotation, resulting in a transformation of a given vertex to a vertex indicated as an out vector direction. Thus a rotation of the +x unit vector **i** ccw about and seen from the +z axis to the position of the +y unit vector **j** is referred to as +*i*. The cw rotation of **i** about +z to the –y axis is –*i*. Rotation of **j** ccw about +x to **k** at +z is +*j*; **j** cw about +x is –*j*; **k** ccw about +y to **i** at +x is +*k*; cw to –x is –*k*. Where two rotations, within semi-colons, are indicated for each vertex, they are not equivalent and are non-commutative. Hence, +*i* and –*k* are the two ½ π rotations capable of taking vertex 1 to the position of vertex 2. The criteria here have been to have an equal number of vectors pointing to and from and total for each vertex. Other scenarios are possible, such as a counter rotational flow for the top and bottom which reverses over time.

Vertex Number	Relative Octant in x,y,z	In Vector from	Out Vector to	Related Rotations viewed from +z, +x &+y axis
1	+x,+y,+z	4;7*	2;5	+*i*,-*k*; +*k*,-*j*
2	-x,+y,+z	1;8*	3;6	+*i*,+*j*; -*k*,-*j*
3	-x,-y,+z	2;5*	4;7	+*i*,+*k*; -*k*,+*j*
4	+x,-y,+z	3;6*	1;8	+*i*,-*j*; +*k*,+*j*
5	+x,+y,-z	8;1	6;3*	+*i*,+*k*; -*l* = -*i* +δ'
6	-x,+y,-z	5;2	7;4*	+*i*,-*j*; -*l* = -*i* +δ'
7	-x,-y,-z	6;3	8;1*	+*i*,-*k*; -*l* = -*i* +δ'
8	+x,-y,-z	7;4	5;2*	+*i*,+*j*; -*l* = -*i* +δ'



A - Unit Cube



B - Monad with Indefinite Expansion

Monad Orthogonality

The asterisks indicate a diagonal vector or flow from the lower to the upper octants, which can also be interpreted as a clockwise rotation out of topological 3-space into geometric 4-space or rather, since it is actually in n-space, the ½ π rotation of the image of an n-form about an n-1 boundary, coupled with the orthogonal translation, δ', through n+1 space to another boundary of the n-form. Note the rotation is the same for each of the four vertices, though the translation is to a different octant in each case. This is consistent with similar constructions in lower n-space, as a square can be constructed by rotating a line segment orthogonally at one end and sweeping it to the diametrically opposite end, and a cube, by rotating a square similarly and translating it to the other side. By "sweeping" we are indicating not simply the translation of, for example, a square, but its extrusion, after copy and rotation, to span and thereby define a 2-cube and its interior. The same can be done with that interior to define a 3-cube and its 4-block, as we might call it.

The figures above show a unit cube constructed according to this table on the left, A, with expansion along one plane of the system shown on the right, B. The result, when coupled with the arrows interpreted as velocity potentials is a horn torus with potential rotation ccw about the upper and lower surface and through the top and into the bottom about the annulus. B can be morphed into a sphere by lengthening the inner diagonals as a spindle torus and into a conventional torus by spreading the top and bottom squares.

While A is presented as a unit form, it can be thought of as a differential form with potential structure, but no inherent size. Size or scale is conferred on it by dropping a unit of inertial density into the center, which then begins to circulate up and out toward and about the upper vertices, at which point angular momentum begins the expansion shown in B, so that instead of going down the four edges, inertia leads to an expansion in the direction of a top surface normal vector and out about the annulus. This is shown as three dimensional, being all that is graphically possible, but it can comprise as many dimensions as are needed to describe it, (di-mension being etymologically to divide or separate with the mind.)

It is the nature of such a configuration that regions more removed from the center will have less inertial density than those more central. Since, as we shall see, gravity is a quantum effect and not a property of inertial density per se, there exists a pressure gradient and Laplacian outward that results in accelerated expansion. If there exists a third derivative over time corresponding with the property of jerk, then we might expect such accelerated expansion to be, not steady, but exponential. Such dynamics create an inertial font at the top of the cube. The bottom of the cube constitutes an inertial sink as the vertices direct the inertial density down and about the periphery, toward a concentration at the center.

The following table outlines the parameters, in which line is understood to mean a bounded line or a line segment.

n-space, element	# in boundary	n-block	surfaces	edges	vertices
0-cube, point	2	l-block, line	0	0	2
1-cube, line	4	2-block, square	0	4	4
2-cube, square	6	3-block, cube	6	12	8
3-cube, cube	8	4-block, hypercube	24	32	16

It may be of some help to provide the algebraic support for these figures. Using the fundamental theorem of calculus, in which the differential *dx* is seen as the point boundary of a line segment, of which there are two, we have the following table for n-cubes, which we can think of as origin centered.

Space	Core = n+1-space		Boundary Order			
n	Open	Closed on Boundary	Volumes	Surfaces	Edges	Vertices
undif	$x^0=dx$		x^3dx^{n-2}	x^2dx^{n-1}	xdx^n	dx^{n+1}
0	x^1	$(x+2dx)^1 =$ $x+2dx$				$2dx$
1	x^2	$(x+2dx)^2 =$ $x^2+4xdx+4dx^2$			$4xdx$	$4dx^2$
2	x^3	$(x+2dx)^3=$ $x^3+6x^2dx+12xdx^2+8dx^3$		$6x^2dx$	$12xdx^2$	$8dx^3$
3	x^4	$(x+2dx)^4=$ $x^4+8x^3dx+24x^2dx^2+32xdx^3+16dx^4$	$8x^3dx$	$24x^2dx^2$	$32xdx^3$	$16dx^4$

Table of n-cube breakdown with n+1-core

With respect to the n-sphere, in which the formulae have been converted to a function of diameter instead of radius, thus relating to the equations above by a factor of $\pi/2(n+1)$, we have the following table. With the exception of the 0-sphere which is effectively a one-half circumference, and this, subject to interpretation, the n-spheres are manifolds without boundary, that is, as manifolds they have no edge. The figures in the edge column do represent an edge of sorts as the extent of their curvature, as it can be seen that they are the formulae for a circle. With respect to the vertices, it can be seen that if we substitute a *dr* for the *dx*, which has some justification at the limit, the formulae of the vertices is that of the n+1-space itself, indicating the single vertex is the center of the space.

Sphere	Ball = n+1-space, $x = 2r$ = the diameter		Boundary Order			
n	Open	Closed on Boundary	Volumes	Surfaces	Edges	Vertices
undif	$x^0=dx$		x^3dx^{n-2}	x^2dx^{n-1}	xdx^n	dx^{n+1}
0	$(\pi/2)x^1$ πr^1	$(\pi/2)(x+2dx)^1 =$ $(\pi/2)x+\pi dx$				πdx πdr
1	$(\pi/4)x^2$ πr^2	$(\pi/4)(x+2dx)^2 =$ $(\pi/4)x^2+\pi xdx+\pi dx^2$			πxdx $2\pi rdx$	πdx^2 πdr^2
2	$(\pi/6)x^3$ $4\pi r^3/3$	$(\pi/6)(x+2dx)^3=$ $(\pi/6)x^3+\pi x^2dx+2\pi xdx^2+(4/3)\pi dx^3$		πx^2dx $4\pi r^2dx$	$2\pi xdx^2$ $4\pi rdx^2$	$4\pi dx^3/3$ $4\pi dr^3/3$
3	$(\pi/8)x^4$ $2\pi r^4$	$(\pi/8)(x+2dx)^4=$ $(\pi/8)x^4+\pi x^3dx+3\pi x^2dx^2+4\pi xdx^3+2\pi dx^4$	πx^3dx $8\pi r^3dx$	$3\pi x^2dx^2$ $12\pi r^2dx^2$	$4\pi xdx^3$ $8\pi rdx^3$	$2\pi dx^4$ $2\pi dr^4$
Table of n-sphere manifold breakdown and n+1-balls						

We have used a cube as a basic form to show the simplicity with which an orthogonal structure can be morphed into an essentially curved structure. Conversely it shows the orthogonal structure that, given any symmetric motion, lies hidden in the most curved of spaces. The horn torus has positive curvature on the outside extent of its annulus were it approaches the configuration of a sphere and negative curvature on the inner confines where is approaches the configuration of a pseudosphere, with a region of flatness in the regions of its upper and lower annular extremal planes. Such regions of flatness, like the crest of a wave, carry the orthogonal potential found at the center outward with expansion, and we might think to find orthogonality in a quantum structure albeit in some sense inverted from the above. Such crest, in this case, represents the transition from negative to positive curvature. If we try to imagine the above cube as a 4-cube, and the monad as of one additional dimension, we might imagine such flatness as the region of our observed universe, the crest of the expanding n-monad.

In the interest of symmetry, it might be imagined that such a structure as A has a conjoined twin that mirrors the dynamics above along the bottom surface with a font from a left hand rotation, so that the center plane becomes an inertial subduction region. We might also envision similar reflection at the top surface, with expansion outward along the central plane. Finally we might imagine a case in which the conjoined system oscillates between extension along axis and plane. We have large scale examples of general toric structure in the collimated jets of active galactic nuclei and the central planes of spiral galaxies, and it would not be surprising to find such on a cosmic level.

The current general cosmological assumption appears to be that spacetime emerged via the big bang from a not quite singular, but probably spherical point. It is also assumed to have been hot. The concept of a thermal birth seems an attempt to explain the presumably conserved energy of the universe initially confined to a very small space. If spacetime itself has inertial properties from which the mass, momentum, energy and power of quantum particles are derived through motion, including its oscillatory activity, then the inertial font attributed to a hot, big bang need not have been either hot or a bang. The total energy of the universe can be contained in a very small, but finite locus of extreme inertial density as a potential energy density as just described.

Unification of the present duality of the general relativistic description of spacetime and the standard model description of quantum matter and its interactions is attempted in a big bang that is a common source for the presumed separate ontologies of both. This holds a paradox, for if gravity is a quantum property, that along with the other interactions, held all together prior to the cosmic inception with a binding intensity that exceeded even the darkest black hole, some yet more energetic mechanism was required to upset a presumed equilibrium and lead to expansion. If matter curves spacetime and in turn conforms to the confines of that curvature, then we have a problem. This is a very short dog with an even shorter tail.

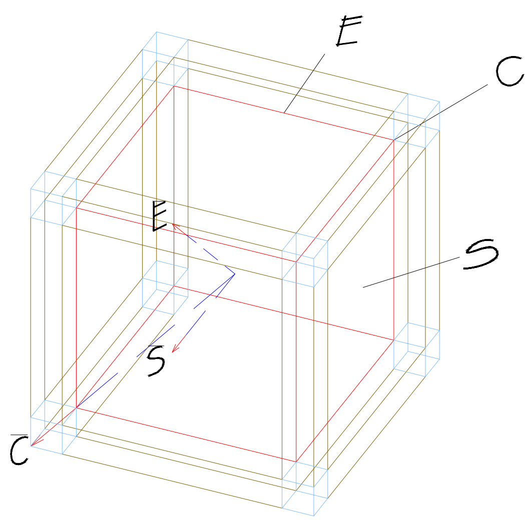
The perspective behind the development herein is that spacetime is an inertial continuum that conforms in some manner to the above description of the monad, oscillating cosmically over time, probably in the manner of a conjoined system. This gives it simply described regions of positive, negative and flat curvature over the cosmic extent, whose local dynamic properties are conditioned by the curvature configuration. As we appear to find ourselves in a generally flat section of this system, we can start our investigation of a quantum structure with an assumption of local Euclidean flat space and see what possible curved structure arises out of an expansive change in that space. We might, for example, find that it appears to have some relationship to A as found at the center of B above after it has been transformed, along the annular surface to a point of isotropy or general flatness.

Isotropic expansion is generally stated as an expansive condition that has no center. In fact, expansion of an n-space has two centers; one is the center of the (n+1)-core for which the n-space forms the cover, and the other is the local center in the n-space from which everything around it is seen to expand. In the customary inflating balloon analogy used to explain isotropic expansion, all points on the surface of the balloon are moving away from the center of the balloon, while any loop drawn on the surface of the balloon will be seen to expand from a point in its center. As in the table concerning the n-spheres above, one vertex at the center end of the radius and the other at any of the circumferential radial end points making the n-sphere’s boundary constitute the centers of expansion. The same holds for the n-cube, and any similarly conforming n-space. Any radial will do. Any expanding space requires simply an extra spatial center, either completely geometric or weighted by some other property, i.e. center of mass, energy, etc. and one (and every) local center. It requires, then, a change in the scale or gauge of only one parameter, **r**, common to each of the n+1 dimensions to effect the isotropic change. The local quantum center is intimately connected to the universal center through an isotropic expansion strain and associated stress.

2 – The Fundamental Inertial Quantum as a Simple Harmonic Oscillator

Isotropic Expansion and the Generation of Rotational Oscillation

We can examine the cube for Space(2) in the above table with the intention of analyzing the effect of an isotropic strain in a condition of assumed uniform dilatation, along with the corresponding stress, on a unit volume of space. In light of the comments just made, we can imagine the center of this cube as one center of expansion and the other as extra dimensional, represented by the indefinite extension of the four diagonals through the eight vertices. We will integrate the differentials to compare the contribution made by each boundary order to the change in the corresponding core, in this case a volume. We are interested in the relative contributions of each order over time to the initial unit volume, *V*, and not to the changing magnitude of the volume itself. We substitute the following boundary placehold identities for Surface, Edge and vertices (Corner), $1^2 S \equiv x^2$, $1^1 E \equiv x^1$, and $1^0 C \equiv x^0$ so as to maintain proper integration. It will be helpful if we assign a "normal" boundary strain vector to each of these components, which in each case will be in the direction in which the boundary is increasing. Thus



Cubic Expansion

$$|\mathbf{S}| = \left|\sqrt{\tfrac{1}{2}}\,\mathbf{E}\right| = \left|\sqrt{\tfrac{1}{3}}\,\mathbf{C}\right| \tag{2.1}$$

$$|\mathbf{E}| = \left|\sqrt{2}\,\mathbf{S}\right| = \left|\sqrt{\tfrac{2}{3}}\,\mathbf{C}\right| \tag{2.2}$$

$$|\mathbf{C}| = \left|\sqrt{3}\,\mathbf{S}\right| = \left|\sqrt{\tfrac{3}{2}}\,\mathbf{E}\right| \tag{2.3}$$

In the following discussion, no assumption is made about the universal configuration or number of dimensions of the space in which the unit cube is embedded. We are only interested, at least initially, in the local geometry, which is assumed to be flat and therefore Euclidean. Thus it is background-independent. As to a fourth spatial dimension, we will see that change in or motion of such dimension is interchangeable with a dimension of time in a three spatial dimension context.

In this case the integration will be simultaneous on each order, as indicated by the pre-subscript *n*, in $\int_n dx^n$ so that we have

$$\int_V dV = 6x^2 \int_0^a {}_1dx^1 + 12x^1 \int_0^a {}_2dx^2 + 8x^0 \int_0^a {}_3dx^3 \tag{2.4}$$

$$\int_V dV = 6S\int_0^a dx + 12E\left(\int_0^a dx\right)\left(\int_0^a dx\right) + 8C\left(\int_0^a dx\right)\left(\int_0^a dx\right)\left(\int_0^a dx\right) \tag{2.5}$$

$$\Delta V = 6aS + 12a^2E + 8a^3C \tag{2.6}$$

Solving for the following ratios, all at unity, where the designations S, E and C are unit names, their dimensional quantities being absorbed in the numerical coefficients of a^n, i.e. 6 square units times *a*, 12 length units times a^2, etc., gives the value of *a* for each equivalence. The ratios have been stated with the highest order in the consequent or denominator so they are decreasing from infinity as *dx* increases, until unity is reached as stated. We have (showing the negative for the sake of symmetry)

$$\frac{S}{E+C} = \frac{6a}{12a^2+8a^3} = \frac{1}{2a+\frac{4}{3}a^2} = 1 \therefore a = -\tfrac{3}{4} \pm \tfrac{1}{4}\sqrt{21} = 0.39564\ldots, -1.89564\ldots \tag{2.7}$$

$$\frac{S}{E} = \frac{6a}{12a^2} = \frac{\frac{1}{2}}{a} = 1 \therefore a = \tfrac{1}{2} = 0.5 \tag{2.8}$$

$$\frac{E}{C} = \frac{12a^2}{8a^3} = \frac{\frac{2}{3}}{a} = 1 \therefore a = \tfrac{2}{3} = 0.66666\ldots \tag{2.9}$$

$$\frac{S}{C} = \frac{6a}{8a^3} = \frac{\frac{3}{4}}{a^2} = 1 \therefore a = \pm\tfrac{\sqrt{3}}{2} = \pm 0.86602\ldots \tag{2.10}$$

$$\frac{E}{S+C} = \frac{12a^2}{6a+8a^3} = 1 \therefore a = \tfrac{3}{4} \pm i\,\tfrac{1}{4}\sqrt{3} = \tfrac{\sqrt{3}}{2}e^{\pm i\frac{\pi}{6}} = 0.86602\ldots e^{\pm i\frac{\pi}{6}} \tag{2.11}$$

$$\frac{S+E}{C} = \frac{6a+12a^2}{8a^3} = 1 \therefore a = \tfrac{3}{4} \pm \tfrac{1}{4}\sqrt{21} = 1.89564\ldots, -0.39564\ldots \tag{2.12}$$

If we think of the cube as embedded in an isotropic elastic continuum, which is of some inertial density and under tension, *dx* represents the work done in displacing or distorting the medium, and by virtue of Gauss' theorem, the integration of that work represents the energy of the distortion. By way of reference, in an ideal elastic medium, the stress operating on the locale is a function of the strain and the elastic modulus as

$$\mathbf{F} = \frac{Y\,\mathbf{E} - 3\sigma\bar{P}\mathbf{1}}{1+\sigma} \tag{2.13}$$

where **F** is the stress tensor, **E** is the strain tensor, *Y* is Young's modulus of elasticity, σ is Poisson's ratio or the negative ratio of lateral to axial or shear to tension strain, $\bar{P}$ is the mean pressure in the medium, and **1** is the idemfactor or unit tensor. Assuming a value of σ of -1/3 for an ideal isotropic 3 dimensional medium we have

$$\mathbf{F} = \frac{3}{2}\left(Y\,\mathbf{E} + \bar{P}\mathbf{1}\right). \tag{2.14}$$

The vector fundamental tension stress component is

$$\mathbf{f} = Y\,\mathbf{e} \qquad (2.15)$$

and is related to the energy distribution by Gauss' theorem for the radial strain

$$E_r = \int_V \nabla \cdot \mathbf{e}_r dv = \oint_S \mathbf{e}_r \cdot d\,\mathbf{S} \qquad (2.16)$$

and Stokes' theorem for the angular or tangential strain

$$E_t = \int_S \nabla \times \mathbf{e}_t \cdot d\,\mathbf{S} = \oint_{\mathbf{r}} \mathbf{e}_t \cdot d\,\mathbf{r}_t \qquad (2.17)$$

These boundary order ratios, then, are inflection points indicating the energy contributions and potential energy gradient changes over time among the boundary components. In an ideal static, kinematic case the change in the ratios with an increase in *dx* would have no functional effect on the components, if *dx* has the same magnitude for each of them as it increases. This would amount to a simple change of scale. The real solutions above would appear to reflect this static condition. However, in a dynamic condition, we might imagine that as each ratio decreases below unity and past the inflection point, the magnitude of the consequent exceeds and affects the antecedent or numerator, whose magnitude then becomes a partial function of the consequent. This would appear to be the case for the complex solutions in particular, which correspond with an angular gradient potential of the boundary vectors from that of the antecedent to the direction of that of the consequent.

These evaluations were done with Maple. It is significant that if we convert (2.11) to complex polar notation as in the last term, the modulus is equal to the value for *a* in (2.10). It is important that we understand that the ratios represent the point at which the change in volume due to the sum totals of all component orders in the antecedent and consequent are equal. It is not the point at which one single component of a given S, E, or C times its appropriate $\int_n dx^n$ is equal to another, since this happens for all at the point where $a = 1$.

In these evaluations, the S component of the strain and hence of the work predominates until (2.7) is reached. At this point, the stress will begin to shift from a predominance of tension to that of shear, meaning there will be a potential for the surface and edge strains to oscillate. As the edges and vertices ring each of the surfaces, the system remains basically stable, however. At the point of (2.8) the edges assume dominance over the surfaces and a gradient is produced for the bulk strain and the tension stress in the direction of the edges. Once again, the 2:1 symmetry of edges to surface maintains stability. At (2.9) the vertices contribute more work than the edges and the strain gradient shifts in their direction. Thus there is a vector potential from the surfaces to the edges to the vertices. One more the symmetry between vertices an edges maintains stability.

Jumping to (2.12), at this point the strain contributed by the vertices dominates both of the other components combined and the related stress is greatest at these locations. This would result in a dissipation of the energy altogether, were it not for the unusual and unique condition created by (2.10) and (2.11). The point at which the strains of the vertices come to equal those of the surfaces is also the point at which their combined strain comes to equal that of the edges, as given by the modulus of the latter's ratio. We can assume that the imaginary component of this ratio indicates a rotational component of $\pi/6$ or 30^{o}, and since the vertices are assuming a predominance over the surfaces at this point, having already exceeded the edge strain, and as there is an imbalance in the number of vertices to surfaces, a necessary break in symmetry ensues.

We can imagine a rotational potential of the surface strain in the direction of the vertices, which by virtue of the asymmetry between S and C, of 3 degrees of rotational freedom and 4 possible rotational axes, results in an eventual rotational strain about one pair of the axes. This is simultaneous with a shift of the Es in the direction of S + C and a dragging of the strains at each of the two axial C poles. This then leads to a rotation of the axial Cs in the direction of one of the three E pairs extending from those two vertices. The equation of (2.11) gives this rotational relationship. The nature of the ambiguous sense in the argument is indicative of the equation of a rotation and its complex conjugate, when viewed from both senses of its axis, i.e. by rotating it about the real axis, where $\pm$ means plus <u>and</u> minus and not plus <u>or</u> minus, if we adjust the Euler identity to

$$e^{\pm i\theta} = \mathbf{sin}\,\theta \pm i\,\mathbf{cos}\,\theta\,. \tag{2.18}$$

One end of the axis of strain then can be shown as indicated by the "symmetry breaking" in (2.21).

$$12a^2E = \left(6aS + 8a^3C\right) \tag{2.19}$$

$$12\left(\tfrac{\sqrt{3}}{2}e^{\pm i\frac{\pi}{6}}\right)^2 E = 6\left(\tfrac{\sqrt{3}}{2}e^{\pm i\frac{\pi}{6}}\right)S + 8\left(\tfrac{\sqrt{3}}{2}e^{\pm i\frac{\pi}{6}}\right)^3 C \tag{2.20}$$

$$e^{-i\frac{\pi}{3}}E = \tfrac{1}{\sqrt{3}}\left(e^{+i\frac{\pi}{6}}S + e^{+i\frac{\pi}{2}}C\right) \tag{2.21}$$

Thus, the strain vector E, rotated in some direction $\frac{\pi}{3}$, is equal to $\frac{1}{\sqrt{3}}$ of the S and C strains rotated $\frac{2\pi}{3}$ in the opposite direction, presumably in the same plane. In fact, this states that C rotates $\frac{\pi}{2}$ while S rotates $\frac{\pi}{6}$. We can see specifically how these rotations occur in Spin Diagrams 1 and 2. We can also see there how a rotation back in time of $\frac{\pi}{3}$ equals one forward in time by $\frac{2\pi}{3}$ and vice-versa, if their plane of rotation, ϕ, is itself rotating at a constant rate with respect to an orthogonal plane, θ, that is where the two axes intersect at the centers of rotation. However, it is shown there that this corresponds with a rotation of θ, back $\frac{\pi}{4}$ and forward $\frac{3\pi}{4}$, indicating a variability in the strain velocity.

It should be understood that this cubic structure is simply an expression of the orthogonal tendency for stress equalization and energy conservation. The condition found at (2.10) and (2.11), then becomes a stable dynamic condition of rotational oscillation or spin, within certain parameters of inertial density and mechanical impedance. If the isotropic tension in this situation was sufficient to increase the strain indefinitely, if the medium was to lose its elasticity and become plastic or even rupture, any tendency to oscillate would be overcome by the transfer of energy via strain to the vertices. Local energy would not be conserved, but be drawn away by the strain.

It is essential to extrapolate this scenario to the hypercube, H, to achieve a full understanding. We will skip the integrals but show the results for the corollary of (2.6) as

$$\begin{aligned}\Delta H &= 8aV + 24a^2S + 32a^3E + 16a^4C \\ &= 1aV + 3a^2S + 4a^3E + 2a^4C\end{aligned} \tag{2.22}$$

There are 25 combinations with corresponding non-ordered permutations or sub-combinations, for the 4-cube; 7 involving all 4 parameters, 12 permutations involving all sub-combinations of 3, and 6 one to one relationships. With the 3-Space, there are 2 single real positive solutions at (2.8) and (2.9), one instance of a complex solution at (2.11), one correspondence between a real and a complex solution at (2.10) and (2.11) where the real value of *a* in one is equal to the complex modulus in the other, and one instance of a correspondence of solutions with sense inversion, (2.7) and (2.12), that is their solutions have the same magnitude, but of opposite sense. As might be expected, the 4-Space shows significantly more of these symmetries. It should be noted that while an attempt has been made to analyze the ratios qualitatively so that all are represented as decreasing with respect to an increasing *dx*, they have not all been checked quantitatively, and some may be increasing as shown. In fact, (2.35) and (2.37) are found to be increasing at the point represented by the first positive solution and decreasing at the second. For (2.32) it is worth stating that for every value of the ratio $0.75 < \left(\frac{S}{V+E}\right) < +\infty$, the modulus is ½ and the argument ranges from 0 to ½ π.

It is important to remember that a given component in the 3-cube is identical to the same component in the 4-cube, but the relationships between them are different. An edge still is bounded by 2 vertices, but there are 4 edges intersecting at each vertex of the 4-cube. A line segment in an *x-y* plane is qualitatively no different than one in the *z-x* or for that matter *z-w* plane. In fact a point in 3-space also has a location in n-space, at least in Euclidean n-space. In the following, it is also important to remember that *a* is not the value of the corresponding ratio, but rather the value found in both antecedent and consequent when the ratio equals 1. The evaluations are based on the following identities in (2.23),

$$V \equiv 1a, S \equiv 3a^2, E \equiv 4a^3, C = 2a^4 \tag{2.23}$$

$$\frac{V}{S}, a = \tfrac{1}{3} \tag{2.24}$$

$$\frac{V}{E}, a = \pm\tfrac{1}{2} \tag{2.25}$$

$$\frac{V}{C}, a = \tfrac{1}{\sqrt[3]{2}}, -\tfrac{1}{2}\left(\tfrac{1}{\sqrt[3]{2}}\right) \ldots \pm i\tfrac{\sqrt{3}}{2}\left(\tfrac{1}{\sqrt[3]{2}}\right) = \tfrac{1}{\sqrt[3]{2}} e^{\pm i\frac{2\pi}{3}} = 0.79370\ldots e^{\pm i\frac{2\pi}{3}} \tag{2.26}$$

$$\frac{S}{E}, a = 0, \tfrac{3}{4} \tag{2.27}$$

$$\frac{S}{C}, a = 0, \pm\sqrt{\tfrac{3}{2}} \tag{2.28}$$

$$\frac{E}{C}, a = 0, 0, 2 \tag{2.29}$$

$$\frac{V}{S+E}, a = -1, \tfrac{1}{4} \tag{2.30}$$

$$\frac{V+S}{E}, a = -\tfrac{1}{4}, 1 \tag{2.31}$$

$$\frac{S}{V+E}, a = \tfrac{3}{8} \pm i\tfrac{1}{8}\sqrt{7} = \tfrac{1}{2}e^{\pm i0.722734248...} \tag{2.32}$$

$$\frac{V}{S+C}, a = 0.31290..., -0.15645... + i1.25436... = 1.26408...e^{\pm i1.694883228...} \tag{2.33}$$

$$\frac{V+S}{C}, a = -1, -0.36602..., 1.36602... \tag{2.34}$$

$$\frac{V+C}{S}, a = -1.36602..., 0.36602..., 1 \tag{2.35}$$

$$\frac{V}{E+C}, -1.85463..., -0.59696..., 0.45160... \tag{2.36}$$

$$\frac{V+C}{E}, a = -0.45160..., 0.59696..., 1.85463... \tag{2.37}$$

$$\frac{V+E}{C}, a = 2.1120..., -0.05604... \pm i0.48331... = 0.48655...e^{\pm i1.686235431...} \tag{2.38}$$

$$\frac{S}{E+C}, a = -2.58113..., 0, 0.58113... \tag{2.39}$$

$$\frac{S+E}{C}, a = -0.58113..., 0, 2.58113... \tag{2.40}$$

$$\frac{E}{S+C}, a = 0, 1 \pm i\tfrac{1}{\sqrt{2}} = \sqrt{\tfrac{3}{2}}e^{\pm i0.615479709...} \tag{2.41}$$

$$\frac{V}{S+E+C}, a = 0.24415..., -1.12207... \pm i0.88817... = 1.43105...e^{\pm i2.472026458...} \tag{2.42}$$

$$\frac{E}{V+S+C}, a = -0.24415..., 1.12207... \pm i0.88817... = 1.43105...e^{\pm i0.669566197...} \tag{2.43}$$

$$\frac{V+E+C}{S}, a = -2.63993..., 0.31996... \pm i0.29498... = 0.43519...e^{\pm i0.744798022...} \tag{2.44}$$

$$\frac{V+S+E}{C}, a = 2.63993..., -0.31996... \pm i0.29498... = 0.43519...e^{\pm i2.396794631...} \tag{2.45}$$

$$\frac{V+S}{E+C}, a = -2.51702..., -0.25673..., 0.77375... \tag{2.46}$$

$$\frac{E+S}{V+C}, a = -0.77375..., 0.25673..., 2.51702... \tag{2.47}$$

$$\frac{V+E}{S+C}, a = 1, \tfrac{1}{2} \pm i\tfrac{1}{2} = \tfrac{1}{\sqrt{2}}e^{\pm i\frac{\pi}{4}} \tag{2.48}$$

Once again using Maple, there are a total of 10 couplings involving complex solutions, of which one is exclusively complex and one other has only a zero for the third and real solution. Only one single real positive solution is given. There are, however, 7 corresponding pairs of solutions involving sense inversion, 5 real and 2 complex. Note that all cases of sense inversion involve a combination of one or more components in either the antecedent and/or consequent and the sense change is associated with a transposition of one or two components in each pair. These do not appear to have any special relationship to the conditions of the 3-cube, at first glance, and we have not investigated them further.

There are several, however, that appear to have a direct relationship to some of the ratios of the 3-cube. Two conditions of correspondence are found between a real positive solution and the complex modulus of a complex solution with a positive real component. (2.28)$\left(\frac{S}{C}\right)$ and (2.41)$\left(\frac{E}{S+C}\right)$ are directly related to (2.10) and (2.11) respectively, the real solution and the modulus of the complex of the second two being equal to the product of the first and $\sqrt{2}^{-1}$. The argument of (2.41) is the angle at the center of a cube between a radial normal to an edge of the cube and one extended along a diagonal to a vertex. (2.25)$\left(\frac{V}{E}\right)$ and(2.32)$\left(\frac{V}{E+S}\right)$ are related to (2.8)$\left(\frac{S}{E}\right)_3$ with a common value for their real solutions and the modulus of the complex one. The cosine of the argument of (2.32) is equal to the solution of (2.27)$\left(\frac{S}{E}\right)_4$, which is the same ratio coupling as (2.8). This pairing (2.32) in turn has a modulus equal to the real and imaginary components of an additional complex solution in (2.48)$\left(\frac{V+E}{S+C}\right)$. This latter solution has an argument of ${}^{\pi}/_{4}$ or 45^{o} which appears to be an extremely stable condition, as found in a sine wave model as the point of maximum power of the wave, where the product of the transverse wave force and transverse wave speed are maximum. It is also the angle of the strain vector E discussed above for the 3-cube, with respect to the plane normal to the spin angular momentum vector as shown in the spin diagrams. In the model developed here, this condition is found to be invariant and rotates about the oscillation's angular momentum vector.

Finally, (2.41)$\left(\frac{E}{S+C}\right)$, (2.48)$\left(\frac{V+E}{S+C}\right)$, and (2.26)$\left(\frac{V}{C}\right)$ are found to be related in a most profound way in the mechanism of the oscillation herein described. The imaginary component of (2.41) equals the modulus of (2.48). Note that (2.26) represents a $\frac{2\pi}{3}$ rotation due to the interplay between the volume and vertex components of strain and a modulus of that strain of $\frac{1}{\sqrt[3]{2}}$. Using the equation for (2.26) or

$$aV = 2a^4C \tag{2.49}$$

$$\frac{1}{\sqrt[3]{2}}e^{\pm i\frac{2\pi}{3}}V = 2\left(\frac{1}{\sqrt[3]{2}}e^{\pm i\frac{2\pi}{3}}\right)^4 C \tag{2.50}$$

tells us that a rotational oscillation of the 4-volume (boundary) strain V of modulus $\frac{1}{\sqrt[3]{2}}$ by $\frac{2\pi}{3}$ is equal to 4 axial rotations about the vertices of the same modulus and argument, where the 2 in the consequent indicates simultaneous rotations of opposite sense at each end of an axis. The oscillation of V is fourth dimensional, and therefore beyond our direct sensory ken, however, the 4 vertices are not, and we can envision the above consequent, the expression in 3 dimension of this four dimensional rotation, as a sequence of 4, $\frac{2\pi}{3}$ rotations about the 4 diagonals of a 3-cube. This sequence leaves the cube unchanged and avoids the entanglement condition, i.e. the continuity of Euclidean 3-coordinates of the cube are not twisted by the sequence. This condition of limits on the twistability of the continuum strain is a necessary consequence of its inertial/elastic properties. As the rotation of V is continuous, we would imagine that the sequence of 4 rotations is continuous, i.e. the strain rotates from one reference diagonal to another about one of the three surface axes of the 3-cube. We can also envision this as one diagonal axis rotating $\frac{2\pi}{3}$, followed by a 2 π rotation of the same sense about one of the adjacent 3-cube surface axes. We can also treat it as a sequence of 4 orthogonal permutations.

We can show this configuration simply. If we align a hyperbolic surface of revolution about the *y* axis of the curve

$$xy = \tfrac{1}{2}\text{, for } x \le \tfrac{1}{\sqrt{2}} \tag{2.51}$$

at each of the eight vertices of a cube so that each of them is at the angle of the argument given by (2.41) as just described, and so that the rims or circles of their bases intersect at the centers of each of the six surfaces of the cube, the following will be found concerning this geometry, which we will call an inversphere. We can also, as an alternative, create a similar construct using a pseudosphere in place of the above surface of revolution. Given a constant negative curvature of -1 for each pseudosphere, the resulting inversphere would have a constant negative curvature.   This points to the development of the monad above. With respect to the inversphere:

1. Each surface of revolution, which we might call a hyper-axis or h-axis and which can be represented by a complex plane, with the imaginary dimension parallel to the circumference of the revolution and the real along the diagonal axis, will have a curvature of negative 1 at the rim, increasing in the negative direction with distance along the asymptote. Here the left four of Figure 4 are shown, their designations corresponding with the axis of Figure 3 below.

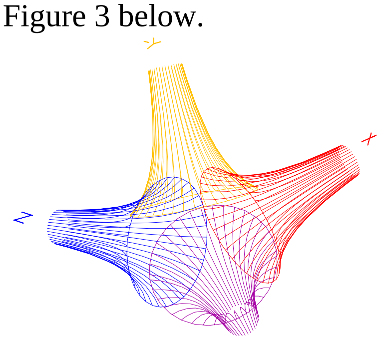


Figure 3

2. The rims will have a radius of $\frac{1}{\sqrt{2}}$. The area of the circle formed by the rim is therefore $\frac{\pi}{2}$, and its complex representation is $\frac{1}{\sqrt{2}}e^{i\theta}$ corresponding with $\left(\frac{V+E}{S+C}\right)$.
3. The rims will intersect orthogonally with each other at the cubic surface centers, so that there are three h-axes adjacent to a given h-axis along the cubic edges which we will refer to as the proximal axes.
4. The rims from h-axes located diagonally across the cubic surface from each other will be parallel or tangential at the same point at which they intersect with their proximal axes. We will call the corresponding parallel axes the distal axes. One set of mutually distal axes can be called the positive h-axes.
5. Each h-axis has a spatial inversion or anti-axis which is proximal to the distal axes of that h-axis. The set of their spatial inversions can be called the negative h-axes.
6. Each rim intersection is a $\frac{2\pi}{3}$ rotation from the others about the cubic diagonal, associating it with $\left(\frac{V}{C}\right)$.
7. The distance between cube surface centers describes an octahedron of edge length $\sqrt{\frac{3}{2}}$. The surface area of the octahedron is therefore $3\sqrt{3}$ and the volume is $\frac{\sqrt{3}}{2}$. The radial normal to the octahedron face is ½.
8. The cube will have an edge measure of $\sqrt{3}$. The surface area of the cube is 18 and the volume is $3\sqrt{3}$.
9. The concentric sphere intersecting at the rim intersections will have a radius of $\frac{\sqrt{3}}{2}$. The surface area is 3π and the volume is $\frac{\sqrt{3}}{2}\pi$.
10. We can think of this arrangement as the expression of a 4-cube in a 3-space, where the orthogonality condition of the 4-D space is met by the rim intersections, the center of each component of sphere, cube, octahedron and h-axis intersections being a common system center.
11. This configuration can be reduced to a 3-space orthogonal system simply by collapsing the cube along the W hyper-axis, as in the figure at left below, resulting in the co-ordinate system at right.

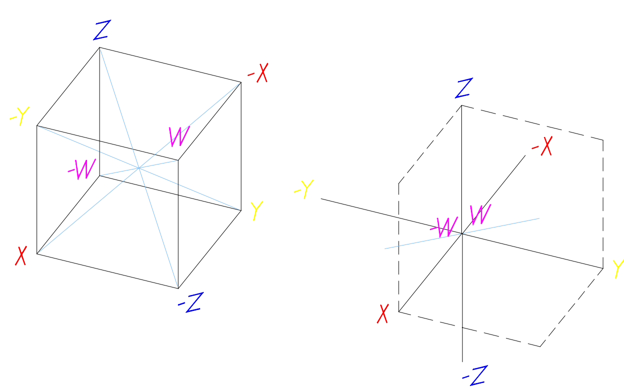


Figure 4

The condition of (2.8)$\left(\frac{S}{E}\right)_3$, (2.25)$\left(\frac{V}{E}\right)$ and (2.32)$\left(\frac{S}{V+E}\right)$ is represented by (2.48)$\left(\frac{V+E}{S+C}\right)$ at each h-axis. Thus the orthogonal projections of the argument of (2.48), described as extending from the system center to each cubic edge midpoint, are equal to the modulus of (2.41)$\left(\frac{E}{S+C}\right)_4$, and the argument of (2.26)$\left(\frac{V}{C}\right)$ is the rotation of that axis between proximal intersections and cubic surface centers.

In terms of $a = \int dx$ we are only interested in positive or increasing real values, although in the context of complex values, some negative real components as in (2.26) are of interest. A deeper analysis would no doubt find significance in all of the couplings, but we are only interested in the general manner in which the 4-cube and the 3-cube couplings might interact. In this regards it is important to remember that in the case of the 4-cube, the volume is a boundary that is increasing while in the case of the 3-cube, it is the base space, held constant, upon which the boundary changes are taking place.

From the perspective of a rotational oscillation, as found in a torsion pendulum or a jump rope oscillation, of interest are those couplings of two boundary parameters, V + E and S+ C, which have an intervening parameter, S and E respectively. More interestingly, in both these cases, V + E for the 4-cube and S + C for both 4-cube and 3-cube, the two-parameter components also have a ratio between themselves whose solution is ($\pm$) real and equal to the modulus of the companion ratio. (2.48) gives the special case of V + E with S + C. Unlike the other three rotational oscillator couplings, it has a positive real solution in addition to its complex solution. It also has the two parameter component ratios in common with the other two oscillators of the 4-cube. The remaining couplings with complex solutions all have intervals between their real and complex moduli solutions, for most exceeding 1, which mitigates against oscillation, with one exception. (2.26)$\left(\frac{V}{C}\right)$ has a real solution that equals its modulus, thereby indicating rotational oscillation. In addition, the cosine of its argument is equal to the modulus and real solution for $\left(\frac{S}{V+E}\right)$ and $\left(\frac{V}{E}\right)$ at $\frac{1}{2}$ and its sine, to the 3-cube modulus and real solution for $\left(\frac{E}{S+C}\right)$ and $\left(\frac{S}{C}\right)$ at $\frac{\sqrt{3}}{2}$, and to the 4-cube modulus and real for $\left(\frac{E}{S+C}\right)$ and $\left(\frac{S}{C}\right)$ at $\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{2}$. Thus the rotational parameters of the other rotational oscillation or spin couplings, can be found in the simple ratio of $\left(\frac{V}{C}\right)$.

Within the context of the 4-cube, the first value that arises is (2.42)$\left(\frac{V}{S+E+C}\right)$ followed closely by (2.30)$\left(\frac{V}{S+E}\right)$. This simply shows that the vertex component adds very little at this juncture, although it does have a rotational element, but the negative real component indicates a significant rotation which would seem out of synch with the small real strain. A similar comment could be made about (2.33)$\left(\frac{V}{S+C}\right)$ which is next in the real order, though the potential rotation is much less. This is followed by (2.24)$\left(\frac{V}{S}\right)$ which has no rotational component. It is significant in that it is the value of Poisson's ratio in an ideal isotropic elastic solid, relating the axial to lateral strain and thereby, tension to shear stress.

Next is (2.35)$\left(\frac{V+C}{S}\right)$ with no rotational component, followed by (2.44)$\left(\frac{V+E+C}{S}\right)$, which has a rotational component. The real solution and therefore the strain is negative, however, and is out of scale with the modulus of the complex solution, which would mitigate against rotational oscillation. This modulus and the positive solution of (2.36)$\left(\frac{V}{E+C}\right)$ are the first values to exceed any of the solutions for the 3-cube. The next ratios (2.25)$\left(\frac{V}{E}\right)$ and (2.32)$\left(\frac{S}{V+E}\right)$ involve the first of the oscillatory groups. The real solution of the first and modulus of the second are equal to each other and to that of (2.8)$\left(\frac{S}{E}\right)_3$, while the cosine of the argument of $\left(\frac{S}{V+E}\right)$ coincides with the real solution of (2.27)$\left(\frac{S}{E}\right)_4$. Thus we might associate an actual oscillation of the 4-cube with the potential $\left(\frac{S}{E}\right)_3$ of the 3-cube. This is followed by (2.39)$\left(\frac{S}{E+C}\right)$ which has a real solution and is the 4-cube corollary of the first ratio of the 3-cube. It is of no special interest other than being, along with (2.9) $\left(\frac{E}{C}\right)_3$ a precursor for the next coupling, which is (2.48) $\frac{V+E}{S+C}$, perhaps the most important of the whole assemblage. Together, $\left(\frac{S}{E+C}\right)$ and $\left(\frac{E}{C}\right)_3$ indicate a growing predominance of E and C over S and then C over E, or shear stress over tension, followed eventually by torsion over shear.

The argument of $\left(\frac{V+E}{S+C}\right)$ represents the power of the strain oscillation, first in the oscillatory twisting of the hyper-axes at $\left(\frac{V}{C}\right)$, then subsequently with the rotational oscillation of the 3-cube itself. Given the above description of the inversphere, the modulus of this solution represents the radius of and in the plane of the rim of the h-axis at the point at which its curvature is -1. The argument is the power phase of an oscillation which can be found as a phase constant in the eventual rotational oscillation of the 3-cube. This is followed by (2.46)$\left(\frac{V+S}{E+C}\right)$, which adds no new oscillatory components, but does show the gaining dominance of the higher order boundary components, E and C. This culminates in a new oscillatory condition at (2.26)$\left(\frac{V}{C}\right)$.

Note that the real value and the modulus of $\left(\frac{V}{C}\right)$ is slightly more than the values of $\left(\frac{V+E}{S+C}\right),\left(\frac{V}{E}\right),\left(\frac{S}{E}\right)_4$ and slightly less than the values of $\left(\frac{E}{S+C},\frac{S}{C}\right)_3$ at oscillation. We can interpret the condition at $\left(\frac{V}{C}\right)$ as an oscillation about each of the 8 vertices. Each oscillation involves a twisting or torsion ultimately of $\frac{2\pi}{3}$ in each direction about each h-axis. The proximal axes will twist counter to the instant rotation sense of a given h-axis as will the anti-axis, all as viewed from the exterior of the system. The distal axes will twist with the same sense as the given h-axis, thus the directional sense of these axes corresponds with their rotational sense vis-à-vis the other axes. The strain on the enclosed sphere at maximum twist will be of a simultaneous lengthening along each <u>cubic</u> axis and flattening in the plane of said axis and the <u>cubic</u> axis from which the strain occurred and at which it is at a minimum, ideally zero, as indicated in the figure below.

The two pairs of distal axes on each surface create two countervailing torques, which in this oscillatory condition are in equilibrium.

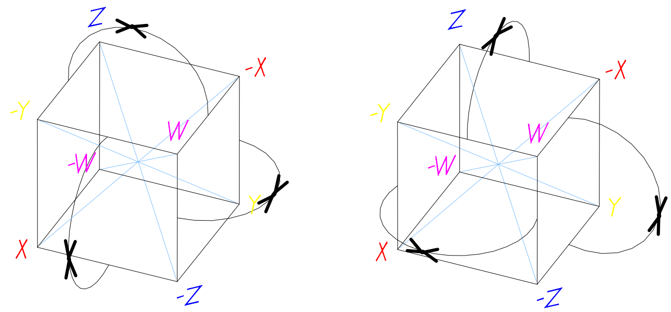


Figure 5

This initial symmetrical condition of $\frac{2\pi}{3}$ rotational oscillation of each of the four diagonals is broken upon *dx* reaching the oscillatory threshold given by $\left(\frac{E}{S+C}, \frac{S}{C}\right)_3$ at $\frac{\sqrt{3}}{2}$. This results in a permanent rotation of $\frac{\pi}{3}$ of one pair of the E vectors as indicated by (2.21), thence the whole system <u>strain</u> continues to <u>oscillate</u>, while the <u>stresses</u> <u>rotate</u> and generate an angular momentum vector. (2.26) indicates the rotation of the stresses in time among and about the four diagonals, which represent the four orthogonal axes of H. The oscillation of the 3-cube is supported and driven by the 4-stress which is concentrated in one transforming axis. (2.48)$\left(\frac{V+E}{S+C}\right)$ represents the power moments or positions of maximum conversion of kinetic to potential energy and vice versa.

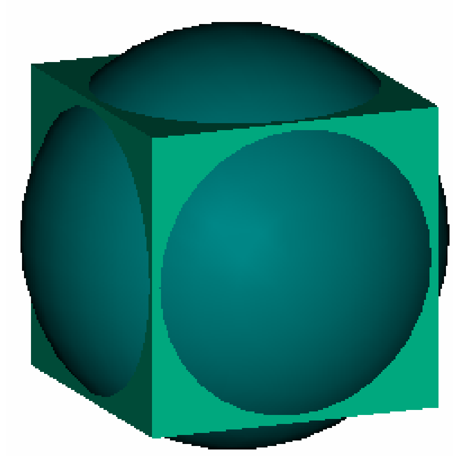
Finally, (2.41)$\left(\frac{E}{S+C}\right)_4$ represents, in addition to the diagonals, a capacitive and an inductive torque that is co-linear with two of the diagonals and is the product of crossing into the power moments from their positions of equilibrium strain and rotates with them about the angular momentum vector, all described later. The modulus and the solution to $\left(\frac{S}{C}\right)_4$ at $\sqrt{\frac{3}{2}}$ represents the radial length from the center of the inversphere and 3-cube to the midpoint of the cubic edge. The solution $a=\sqrt{\frac{3}{2}}e^{\pm i0.61547...}$ in this case indicates a rotation of this vector into the diagonal or one of the h-axis or of **E** into **C**. Solving for $\left(\frac{E}{S+C}\right)_4$

$$4a^3E = 3a^2S + 2a^4C$$
$$4\left(\sqrt{\tfrac{3}{2}}e^{\pm i0.61547...}\right)^3 E = 3\left(\sqrt{\tfrac{3}{2}}e^{\pm i0.61547...}\right)^2 S + 2\left(\sqrt{\tfrac{3}{2}}e^{\pm i0.61547...}\right)^4 C \tag{2.52}$$

after reduction and some parsing gives

$$2\left(\sqrt{\tfrac{3}{2}}e^{-i0.61547...}\right)E = \left(\sqrt{\tfrac{3}{2}}e^{i\frac{\pi}{2}}\right)^2 S + \tfrac{3}{2}e^{+i1.23095...}C. \tag{2.53}$$

Here as with the companion relationship for the 3-cube, we have “broken symmetry” with the rotational senses, and see that rotation of two edge strains into an adjacent corner is equal to two orthogonal rotations of a surface strain and a flip of a vertex strain from one h-axis to a proximal axis. The moduli in this case correspond to the metrics of the inversphere, where $\frac{3}{2}$ is the distance from the cubic center to the cubic vertex. We will see that this represents an instance of beta decay, where the surface and vertex rotations indicate the flip of the electrical phase torques from one pair of vertices to one of three proximal pairs. In the case of the inductive torque, we have an electron emission along with a flip of the magnetic moment, and in the case of the capacitive torque, we find a positron emission, without the magnetic moment flip.



Equal Area Cube and Sphere

The position indicated as the midpoint on the cubic edge is of special interest. If we analyze a concentric cube and a sphere of equal surface area, and presumably of equal total surface stress, we will find that the radial to the midpoint as represented by $\left(\frac{S}{C}\right)_4$ exceeds the spherical radius (and the path of the rotational oscillation strain) at that point by a factor of

$$\delta r = \tfrac{\sqrt{2\pi}}{3}\left(\sqrt{\tfrac{3}{2}} - \tfrac{3}{\sqrt{2\pi}}\right) = \sqrt{\tfrac{\pi}{3}} - 1 = 0.023326708\ldots. \tag{2.54}$$

This indicates that the rotational path of the strain constricts the diagonals and restricts the operation given by $\left(\frac{E}{S+C}\right)_4$. Thus this differential must be overcome by the increase in stress of that operation. If we assume that the differential given by (2.54) is one component of the cross sectional area on which an orthogonal stress is operating, then the square of that value gives a differential stress required for the diagonal to flip of

$$\delta r^2 = 0.0005441353061\ldots. \tag{2.55}$$

The ratio of differential stress to the augmented total is then

$$\frac{\delta r^2}{1+\delta r^2} = \frac{0.0005441353061}{1.0005441353061} = 0.0005438393841\ldots \tag{2.56}$$

which when inverted is

$$\frac{1+\delta r^2}{\delta r^2} = 1838.778193 \tag{2.57}$$

It bears noting that the 2002 CODATA ratio of the electron to neutron mass is 0.00054386734481(38), or within 2.796…x10^{-8} of the value of (2.56). Thus the ratio of the differential stress needed to produce beta decay and the stress of fundamental oscillation correlates significantly with the ratio of the mass-energy of the product of that decay, the electron, and that of the fundamental oscillation, the neutron.

With reference to beta decay, one additional observation concerns the weak mixing angle yielded by the final measured asymmetry stated in the September, 2005 issue of Physics Today by Bertram Schwarzschild in "Tiny Mirror Asymmetry in Electron Scattering Confirms the Inconstancy of the Weak Coupling Constant" as $\mathbf{sin}^2\,\theta_W = 0.2397 \pm 0.0013$. If we consider the surface area of a sphere in steradians as 4π, the portion spanned by each cubic edge in conjuction with the above development is one twelfth that or an area of $\pi/3$. A linear component of that measure would therefore be $\sqrt{\frac{\pi}{3}}$ and would correspond generally and perhaps in some statistical manner with the distance from a cubic surface vector to a vertex vector as in the interplay between S and C at $\left(\frac{E}{S+C}, \frac{S}{C}\right)_{3,4}$. The arc distance between the mid-point of that arc and each of the three parameters E, S, and C is then ½ $\sqrt{\frac{\pi}{3}}$. We then have the following, which is stated phenomenologically and without causal analysis

$$\mathbf{sin}^2\left(\tfrac{1}{2}\sqrt{\tfrac{\pi}{3}}\right) = 0.239735827\,. \tag{2.58}$$

This is the last ratio of interest, as it marks the final oscillatory condition for the 4-cube. We can show this development in the following orthogonal matrices. First the above rotational oscillation can be given by a 3-D strain spin matrix, $\mathrm{E}_{\mu\nu}$, in which we assume a stationary spin angular momentum vector, $S_L\left(=\hbar\right)$, as our reference frame pointed in the $+z$ direction. We will give all six semi-axes, where a/b^{-1} is the direction of strain given as the double dot product of a into the b surface of the cube, as developed in Physics of Waves, Elmore and Heald, or

$$\begin{aligned}\mathrm{E}_{\mu\nu} &= \pm\,\mathbf{A:B} = \pm\sum_{i=1}^{3}\sum_{j=1}^{3} a_{ij}b_{ij}\\ &= \left(a_{xx}b_{xx} + a_{xy}b_{xy} + \mathbf{...} + a_{zz}b_{zz}\right) + \left(a_{-x-x}b_{-x-x} + a_{-x-y}b_{-x-y} + \mathbf{...} + a_{-z-z}b_{-z-z}\right)\\ &= 2\left(a_{xx}b_{xx} + a_{xy}b_{xy} + \mathbf{...} + a_{zz}b_{zz}\right)\end{aligned} \tag{2.59}$$

$$\mathrm{E}_{\mu\nu} = \begin{pmatrix} x/x & y/x & z/x \\ x/y & y/y & z/y \\ x/z & y/z & z/z \end{pmatrix} + \begin{pmatrix} x/{-x} & y/{-x} & z/{-x} \\ x/{-y} & y/{-y} & z/{-y} \\ x/{-z} & y/{-z} & z/{-z} \end{pmatrix} \tag{2.60}$$

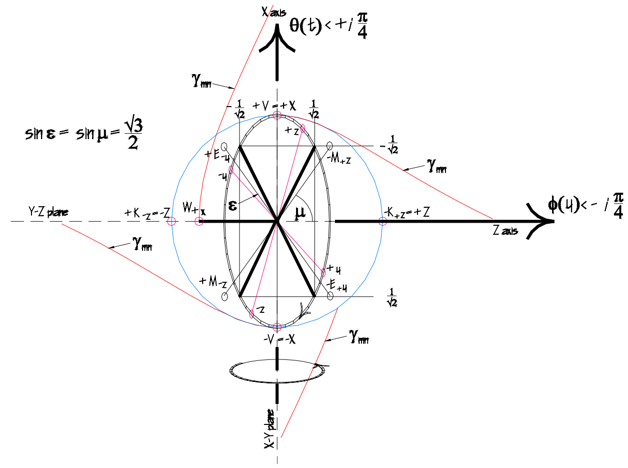
where a unit statement of a is

$$\begin{pmatrix} 1 & 1 & \mathbf{sin}\,\omega t \\ -1 & 1 & -\mathbf{cos}\,\omega t \\ -\mathbf{sin}\,\omega t & \mathbf{cos}\,\omega t & 1 \end{pmatrix} + \begin{pmatrix} -1 & -1 & -\mathbf{sin}\,\omega t \\ 1 & -1 & \mathbf{cos}\,\omega t \\ \mathbf{sin}\,\omega t & -\mathbf{cos}\,\omega t & -1 \end{pmatrix} \tag{2.61}$$

Note that plugging (2.61) into (2.60) for unit values of b gives

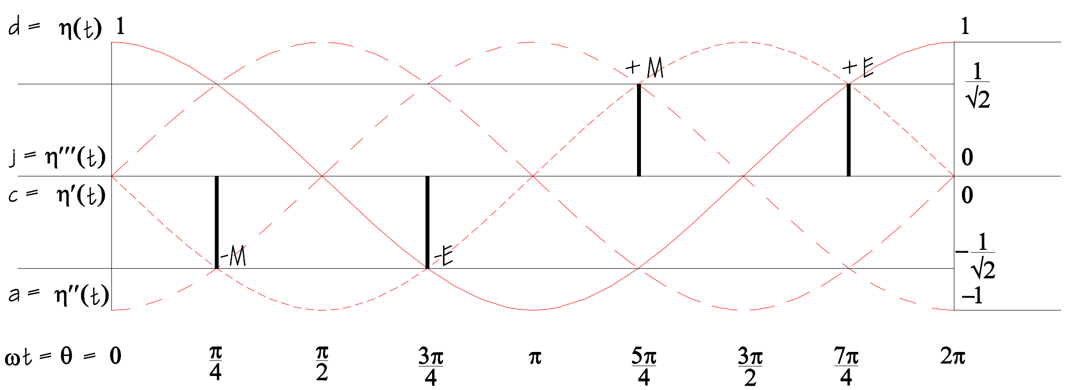
$$2\begin{pmatrix} 1 & 1 & \sin\omega t \\ -1 & 1 & -\cos\omega t \\ -\sin\omega t & \cos\omega t & 1 \end{pmatrix} \tag{2.62}$$

The following Wave Diagrams show the conditions brought about by the above development.



Wave Diagram 1 – Double Rotation, ϕ (θ)

The thin curved red lines represent the strains associated with $\left(\frac{V+E}{S+C}\right)$ where the argument of that coupling solution is found in the dark central cross representing the power moments, charge/potential (E) and induction/kinetic (M), represented by the ϕ vector, and $\left(\frac{V}{C}\right)$ which has rotated the x axis from its initial position, X. These power moments are analogous to the positions of maximum power of a wave on an ideal stretched string shown below.

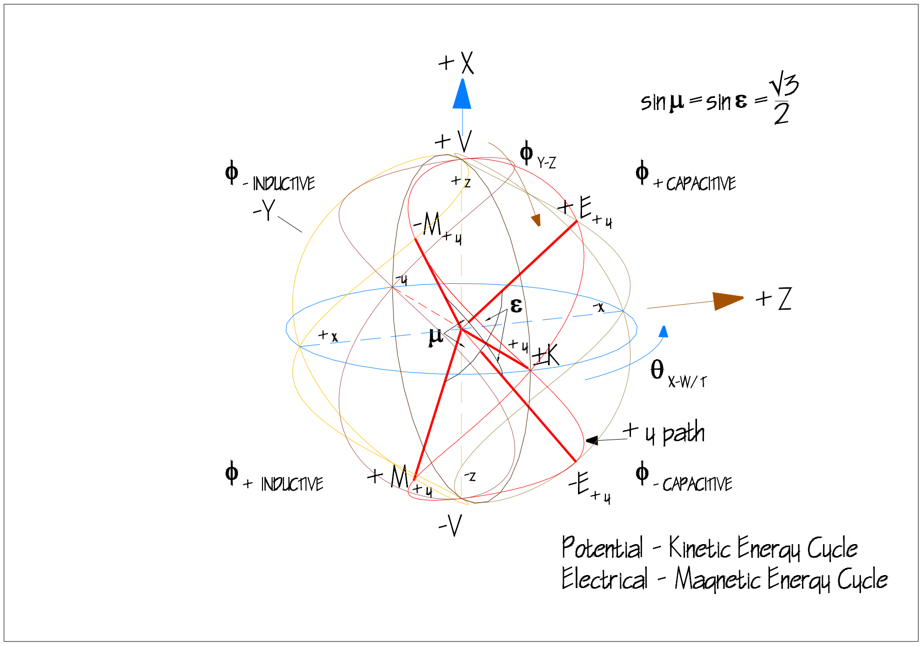


Wave Diagram 2 – Kinematic Functions of η(t)

The following Spin Diagram shows the strain path in red for point +y at $\theta = \omega t = n2\pi$ at which time the strain at that point is in a relative equilibrium position, (other than a ½ π twist) and transforming so that

$$x/y = -1,\ z/y = -\cos n2\pi = -1 \tag{2.63}$$

Note that this recapitulates Figure 2.



Spin Diagram 1 – Spin Energy Cycle

The initial condition consists of a symmetrical 4-oscillation given by $\left(\frac{V}{C}\right)$ at all h-axes. The initial co-ordinates are given by the upper case +*X*, +*Z*, and –*Y*. The oscillation about the (+*X*, +*Y*, +*Z*)-(-*X*,-*Y*,-*Z*) h-axis, in which +*x* oscillates from its equilibrium position through +*X*, between +*Y* and +*Z*, is broken at $\left(\frac{E}{S+C}\right)_3, \left(\frac{S}{C}\right)_3$ as indicated by (2.21), when +*x* is at +*Y* by a clockwise rotation about the +*X* axis to arrive at the position in the spin diagram. From this point the restorative forces begin the rotational oscillation in a counterclockwise sense about +*X* and the spin vector, S_L.

The heavy red *E* and *M* moments show the points of maximum conversion of kinetic to potential energy, *E*, and potential to kinetic energy, *M*, for that locus of strain, and are separated in time by $\theta = \omega t = \frac{\pi}{2}$. The angles ε and μ are $\frac{\pi}{3}$, ((2.11)and (2.21)$\left(\frac{E}{S+C}\right)_3$). The angles between the points on the path, at *E* and *M* and the plane, θ, at the midpoint of the line from the system origin or center to the point of maximum kinetic energy, *K*, are 0.615479709 (see (2.41)$\left(\frac{E}{S+C}\right)_4$) and between *M* and *E* moments and the plane, θ, at the center is $\frac{\pi}{4}$ (see (2.48)$\left(\frac{V+E}{S+C}\right)$).

Absent nuclear or other inertial confinement, S_L is not constrained to +X or any other particular direction over time. The rotational oscillation continues at resonant frequency until the conditions found at $\left(\frac{E}{S+C}\right)_4, \left(\frac{S}{C}\right)_4$ precipitate a flip in the M moments and their corresponding inductive torques.

The symmetry between the 3-cube and the 4-cube can be represented by the following orthogonal matrices of space/time permutations involved in the above description when summed over one cycle. As there is inversion symmetry, only half the matrix is shown. The permutations indicate the physical distortion of the medium. The 0 time given below is at the start of an arbitrary cycle and corresponds with $\theta = \omega t = \pi$ in the diagram above (for reason having to do with the history of the generation of the graphics.)

This series of permutations can be carried out by a sequence of ccw $\frac{2\pi}{3}$ rotations about the cubic diagonals as with $\left(\frac{V}{C}\right)$ given by the combination of each diagonal's defining cube sense/axes, facing the cubic center. There is an obvious lack of symmetry in this matrix between all the axes, since $+y$ and $+z$ apparently oscillate over one π, while $+x$ rotates around +z. Note that at any phase, when either $+y$ or +z is at the equilibrium or <u>m</u>id point of its oscillation, the other axis, +z or $+y$, is at the <u>e</u>xtremum. This can be seen if we were to start our phases at ω^1. In this case, however, in the rotation column it appears that $+y$ is rotating about +z, and that it is $-z$ and $-x$ that are at e and m respectively. The e and m in the table below merely indicate the condition at $\omega t = 0$, and they alternate with each $\pi/2$ change.

	Phase	rotation	m	e	ccw at
ω^0	$\omega t = 0$	$+x$	$+y$	$+z$	
ω^1	$\omega t = \pi/2$	$+y$	$-z$	$-x$	$-x,-y,+z$
ω^2	$\omega t = \pi$	$-x$	$+y$	$-z$	$+x,-y,+z$
ω^3	$\omega t = 3\pi/2$	$-y$	$+z$	$-x$	$+x,+y,+z$
ω^4	$\omega t = 2\pi$	$+x$	$+y$	$+z$	$-x,+y,+z$

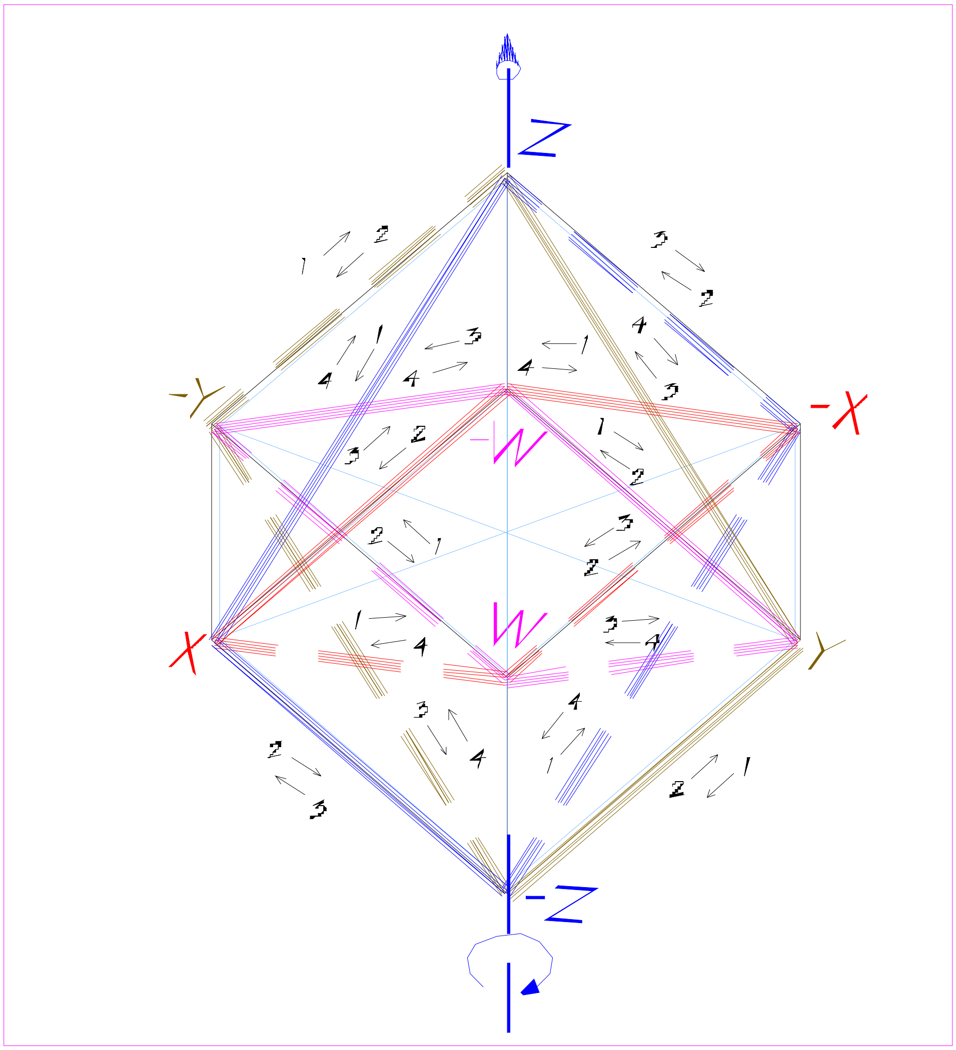
In actuality, closer analysis shows that both $+y$ and $+z$, as strains, oscillate between $+X$ and $-X$, through their initial positions, $+Y$ and$+Z$, while $+x$ and $-x$ rotate as <u>sustained strain and stress</u> points in the Y-Z plane, twisting so as to maintain continuity with their intital positions at $+X$ and $-X$.

We can create a corresponding 4-D orthogonal permutation matrix, adding a fourth dimension, w, and representing it with the inversphere.

	Phase	e	m	e	m	ccw at cube
ω^0	$\omega t = 0$	$-x$	$+y$	$+z$	$-w$	
ω^1	$\omega t = \pi/2$	$-w$	$-z$	$+x$	$+y$	$+x/-y$ edge
ω^2	$\omega t = \pi$	$+x$	$+y$	$-z$	$-w$	-x,+z,-y,+w face
ω^3	$\omega t = 3\pi/2$	$-w$	$+z$	$+x$	$-y$	$-x/+y$ edge
ω^4	$\omega t = 2\pi$	$-x$	$+y$	$+z$	$-w$	+x,-z,+y,-w face
Results in a 4-D rotation ccw at z/-w edge						

The tie in between the 3-D and 4-D matrices follows, but first we should note a few properties. First, there is no sustained rotation about a diagonal axis, though there appears to be one analogous to that of the 3-D at one of the edges, and all permutations are shown to be oscillations between two ends of an axis through an intermediary axis. These oscillations can be generally described by a rotation of an axis not orthogonal to the other

4 which itself rotates about the center of the cube and through the plane bisecting the cube at the $+x,+y,-x,-y$ vertices. The transformation between the ω^3 and $\omega^4 = \omega^0$ phase is analogous in three dimensions to a spatial inversion through the origin or center of the cube followed by a ccw 90^o rotation at the bottom face.



Sum of Oscillations Produces Rotation
1 - CCW at X/-Y Edge
2 - CCW at Top Face
3 - CCW at -X/Y Edge
4 - CCW at Bottom Face

As to the correspondence between the 3 and 4-D matrices, to remove the *w* dimension, we can start with the 4-D form and make the equations shown in the left column, abstracting their correspondence from the matrix.

	Phase	e	m	e	m
$\omega^0 = -w$	$\omega t = 0$	$-x$	$+y$	$+z$	
$\omega^1 = -w$	$\omega t = \pi/2$		$-z$	$+x$	$+y$
$\omega^2 = -w$	$\omega t = \pi$	$+x$	$+y$	$-z$	
$\omega^3 = -w$	$\omega t = 3\pi/2$		$+z$	$+x$	$-y$
$\omega^4 = -w$	$\omega t = 2\pi$	$-x$	$+y$	$+z$	

We then combine the first and last columns to get a table that is essentially the same as the 3-D form above with the important exception of the senses of the *x*s and the ambiguous nature of the first column.

	Phase	e&m	m	e	
$\omega^0 = -w$	$\omega t = 0$	$-x$	$+y$	$+z$	
$\omega^1 = -w$	$\omega t = \pi/2$	$+y$	$-z$	$+x$	
$\omega^2 = -w$	$\omega t = \pi$	$+x$	$+y$	$-z$	
$\omega^3 = -w$	$\omega t = 3\pi/2$	$-y$	$+z$	$+x$	
$\omega^4 = -w$	$\omega t = 2\pi$	$-x$	$+y$	$+z$	

We see, however, that this is associated with a contrary sense of the *w* in the frequency column. Inverting this sense is equivalent to a time reversal. Transposing this sense and converting the first oscillation denoted with an e&m to a rotation, we have the original. The underlying symmetry between the 3-D fundamental quantum <u>rotational</u> oscillation and a 4-D spatial oscillation becomes apparent.

	Phase	rotation	m	e
$\omega^0 = +w$	$\omega t = 0$	$+x$	$+y$	$+z$
$\omega^1 = +w$	$\omega t = \pi/2$	$+y$	$-z$	$-x$
$\omega^2 = +w$	$\omega t = \pi$	$-x$	$+y$	$-z$
$\omega^3 = +w$	$\omega t = 3\pi/2$	$-y$	$+z$	$-x$
$\omega^4 = +w$	$\omega t = 2\pi$	$+x$	$+y$	$+z$

Two items should be mentioned concerning this development with respect to the standard model. The first concerns the manner in which this description might be consistent with the quark model. It is obvious that there is an internal spin structure in the nature of the nodes, antinodes, and various moments and torques of the wave strain. We will see that an analysis of these features reveals a fractional charge, and that the phenomenology of quark confinement is the ontology of wave nodes and antinodes. The second concerns the other two flavors of particle families, generally centered around their leptons, the tau and muon. Since there is strong evidence that these last two particles mutate, specifically in the case of solar radiation, the assumption here is that they are relativistic products of beta decay from the same fundamental rotational oscillation, the neutron.

Dynamic Functions

To understand the dynamic functions of the above oscillation, we can start by examining the functions of an ideal string in "jump rope" oscillation, i.e. a standing wave of one half wavelength, shown below. The subscript noughts in the parameters indicate their fundamental characteristic values, which are properties of the resonance of the wave bearing medium. The oscillation in this case is of Simple Harmonic Motion, so there are no harmonic overtones traveling along the string, which is of some linear inertial density, λ_0, and some tension stress force, τ_0, given by

$$\tau_0 = f_0 \cdot \mathrm{A}_0 \tag{2.64}$$

where f_0 is the tension stress and A_0 is the unit cross sectional area. This is related to the resonant angular frequency, ω_0, and commensurate angular wave number, κ_0, and velocity of the strain oscillation, c_0, as

$$\lambda_0 = \frac{1}{c_0^2}\tau_0, \text{ where } c_0 = \frac{\omega_0}{\kappa_0} = \frac{\partial\theta}{\partial t}\frac{\partial x}{\partial\theta} = \frac{\partial x}{\partial t} \tag{2.65}$$

When normalized,

$$c_0 = \frac{\partial x_0}{\partial t_0} = \frac{\partial t_0}{\partial x_0} = 1 \tag{2.66}$$

and in an isotropic space, for a unit value of x_0,

$$|x_0| = |\hat{\mathbf{r}}| = r_0 \tag{2.67}$$

Therefore, what may not be so obvious, but assuming the units of distance r, in A and in κ are the same, for $\theta = 1$,

$$\lambda_0\omega_0^2 = \tau_0\kappa_0^2 = f_0 \tag{2.68}$$

Also, of eventual interest, the mechanical impedance, Z_0, of the string is

$$Z_0 = \lambda_0 c_0 = \frac{\tau_0}{c_0} \tag{2.69}$$

and the power, P_0, transported by the wave, if it is a traveling wave, and hence retained if it is standing is

$$P_0 = \lambda_0 c_0^3 = Z_0 c_0^2 = \tau_0 c_0 \tag{2.70}$$

We can represent the oscillation with the Euler identity, using both the real and imaginary parts, as a complex standing wave, ϕ, where $\theta = \kappa x \pm \omega t$, and $\kappa x = 0$,

$$\phi = \eta + \varsigma = A(\mathbf{cos}\,\theta \pm i\,\mathbf{sin}\,\theta) = Ae^{i\theta} \tag{2.71}$$

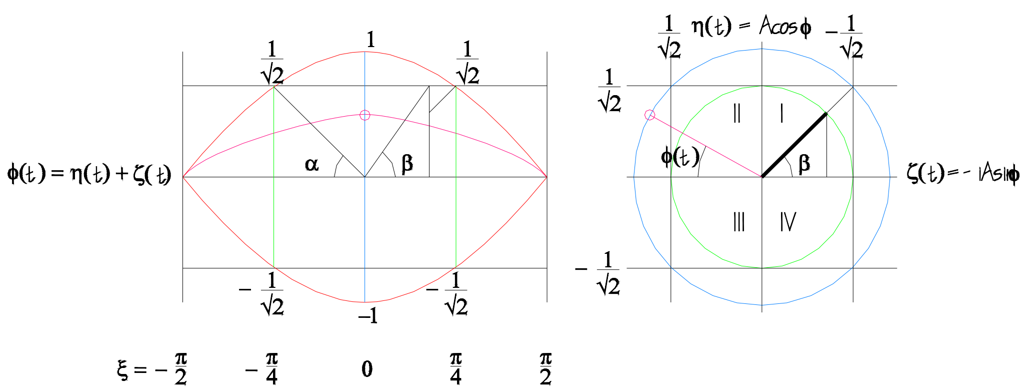
so that for any time, t, and where A is a real amplitude, (not the cross-sectional area A_0 above) equal to the maximum radius of the string path and modulus of the complex polar form, r,

$$\begin{aligned}\eta &= A\,\mathbf{cos}\,\omega t\\ \varsigma &= iA\,\mathbf{sin}\,\omega t\end{aligned}. \tag{2.72}$$

The ambiguity of the rotational sense is once again used, since in the case of a conservative or closed SHM where there is no damping or loss of energy of the system, time is cyclical and reverses with each half cycle of the oscillation. This also reflects the fact that what is clockwise on one side of the path is counterclockwise on the other, the latter of which is shown below.

We will stipulate that the phasing of ϕ and θ remain synchronized so that at time $t = 0$, the oscillation is at the top of the cross section on the right, at $\eta = A$. We can imagine that the cross section of the string, indicated by the small round circle at the end of the path radial, echos the path shown in blue below, maintaining the same orientation as the path, and if we imagine the path to represent the string cross section, that a density gradient resulting from the differential stretching of the string points along the radial from the small red circle through the center of the blue to its opposite side in the direction of increasing density. Thus, a two dimensional entity confined in awareness to the cross section and whose orientation is locked in any arbitrary y direction, parallel to η, would notice a rotation of the density gradient and might think that it is he that is rotating or spinning about with respect to the direction of the gradient. Thus to him the stress rotates while the strain oscillates in two orthogonal dimensions. A one dimensional being would simply feel the stress oscillate between two poles.

We can next think of the cross section of the string as reaching toward infinity, so that the displacement of the string is instead seen to be a lateral or transverse strain of the continuum in oscillation in two dimensions from and about its position of rest at the center of the path.



Wave Diagram 3 – Simple Harmonic Oscillation of ϕ

The usual kinematic functions of the oscillation can be given for ϕ, just as it was for η in Wave Diagram 2 above, assuming the ccw rotation as shown above, as

Displacement, r $$\phi(t) = Ae^{i\omega t} \tag{2.73}$$

Velocity, c $$\phi'(t) = i\omega Ae^{i\omega t} \tag{2.74}$$

Acceleration, a $$\phi''(t) = -\omega^2 Ae^{i\omega t} \tag{2.75}$$

Jerk, j $$\phi'''(t) = -i\omega^3 Ae^{i\omega t} \tag{2.76}$$

Due to our stipulation of conservation of energy, in a normalized system, where $d\omega = 0$ and $\omega = 1$, we have the following equivalence of the integral, $\Phi(t)$, and j

$$\Phi\left(t\right) = \phi'''\left(t\right) = -i\omega^3 Ae^{i\omega t} = \frac{Ae^{i\omega t}}{i\omega} \tag{2.77}$$

Their dynamic counterparts are:

Constant of Inertia, ת(tav) $\quad {}^0_0\text{ת}^0_0 \equiv m\phi\left(t\right) = mAe^{i\omega t} = \text{ת}$ (2.78)

Transverse momentum, G_0 $\quad \text{ת}_1 \equiv m\phi'\left(t\right) = m\left(i\omega Ae^{i\omega t}\right) = i\text{ת}\omega$ (2.79)

Transverse wave force, τ_0 $\quad \text{ת}_2 \equiv m\phi''\left(t\right) = m\left(-\omega^2 Ae^{i\omega t}\right) = -\text{ת}\omega^2$ (2.80)

Transverse wave yank, Y_0 $\quad \text{ת}_3 \equiv \text{ת}^1 \equiv m\phi'''\left(t\right) = m\left(-i\omega^3 Ae^{i\omega t}\right) = -i\text{ת}\omega^3 = \dfrac{\text{ת}}{i\omega}$ (2.81)

Some explanation is in order. Given a fixed ω_0 according to (2.65) and a fixed m, it is apparent that these functions are invariants of the system. (2.80)appears to be an expression of Hooke's law of force for simple harmonic motion generally given as

$$F = -\left(m\omega^2\right)\eta\left(t\right) = -k_{spring}\eta\left(t\right), \ \therefore \ -k_{spring} = -\left(m\omega^2\right) \tag{2.82}$$

where the spring constant, k_{spring}, includes a constant value for the mass, m. In this case, ϕ is substituted for η, with its magnitude a constant as a complex modulus, its instant direction given by the argument. Thus it is a scalar invariant, as is τ_0. Since the only difference between (2.80) and the other three functions are the powers of ω_0 and the $\frac{\pi}{2}$ directional change given by i, they are themselves all scalar invariants under rotation and translation. Since $A = r$, and since the displacement of r in ϕ_{yz} is normal to the direction of κ_x for any time t, the imaginary sense is entered, (2.78) becomes

$$\text{ת} = mr = \frac{m}{\frac{\partial\theta}{\partial r}} = \frac{m}{i\kappa} = \frac{m_0}{i\kappa_0} \tag{2.83}$$

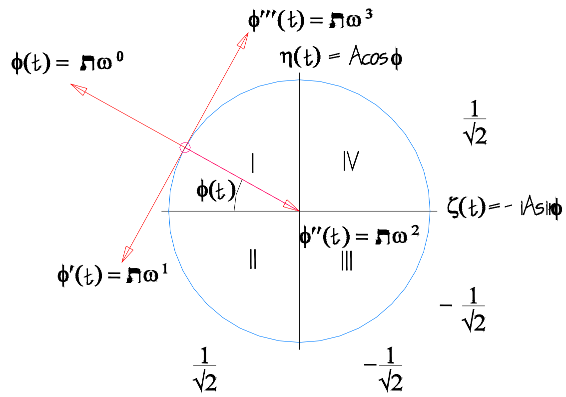
and

$$m_0 = i\text{ת}\kappa_0 = \frac{\text{ת}}{Ae^{i\kappa x}} \tag{2.84}$$

which we can substitute the last term back into (2.79), (2.80), and (2.81), resulting in the final term of each. For an arbitrary fixed reference frame, $Ae^{i\omega t}$ varies sinusoidally, but for a frame rotating at ω_0, with the complex modulus, is constant and at unity when divided by $Ae^{i\kappa x}$.

Thus mass is essentially the inverse wave modulus and the transverse wave number. (This latter should not be confused with what we might call the "amplitude wave number", $A^{-1} = r^{-1}$, which for the fundamental oscillation is the same as the wave number. Thus the neutron is an essentially spherical wave form, while the electron can be modeled in one of two alternatives as an extremely thin prolate spheroid wave form transmitted by the central oscillation at beta decay, whose major axis, $\pi\kappa_e^{-1}$, exceeds its minor axis, r_e, essentially by their ratios of 1.334775525…x10^6. The neutron is then transformed into an oblate spheroid wave form of the proton, whose major axis exceeds its minor by a factor of 1.000000895… . This will be shown later)

The left hand identity of each is a compact and convenient notation for each invariant, ${}_0^0\sqcap_0^0$, where the subscript indicates complex differentiation and the superscript indicates complex integration, the right hand side with respect to time and the left hand side, not yet used, for the same calculus with respect to space. We can show these invariant functions graphically, and with the sense omitted, as



Wave Diagram 4 – Dynamic Functions of φ (t)

The above discussion on the derivation of the wave form indicated that the rotational oscillation resulted after a $\frac{2\pi}{3}$ rotation about one of the h-axes with $\left(\frac{V}{C}\right)$, which results in an orthogonal rotation of all six cubic axes. This is reflected in the imaginary sense of (2.83). Thus we can differentiate with respect to space and get the following. As with the rotations with respect to time, the rotations of space about the h-axes with $\left(\frac{V}{C}\right)$ are cyclic, so that the first order of integration with respect to space is equal to the third order of differentiation.

Constant of Inertia, $\sqcap$ $\qquad {}_0^0\sqcap_0^0 \equiv m\phi(x) = \frac{\sqcap}{Ae^{i\kappa x}}Ae^{i\kappa x} = \sqcap$ (2.85)

Oscillation mass, m_0 $\qquad {}_1\sqcap \equiv m\phi'(x) = \frac{\sqcap}{Ae^{i\kappa x}}\left(i\kappa Ae^{i\kappa x}\right) = i\sqcap\kappa$ (2.86)

Linear inertial density, $\lambda_0\,{}_2\sqcap \equiv m\phi''(x) = \frac{\sqcap}{Ae^{i\kappa x}}\left(-\kappa^2 Ae^{i\kappa x}\right) = -\sqcap\kappa^2$ (2.87)

Moment of inertia, $I_0\,{}_3\sqcap \equiv {}^1\sqcap \equiv m\phi'''(x) = m\Phi(x) = \frac{\sqcap}{Ae^{i\kappa x}}\left(-i\omega^3 Ae^{i\kappa x}\right) = -i\sqcap\kappa^3 = \frac{\sqcap}{i\kappa}$ (2.88)

We can complete this picture by creating an orthogonal 4x4 matrix of the functions, using the inertial constant notation, in the second iteration of which we substitute integration for the third order of differentiation for both time and space, making .

$$\begin{bmatrix} {}_0\text{ח}_0 & {}_0\text{ח}_1 & {}_0\text{ח}_2 & {}_0\text{ח}_3 \\ {}_1\text{ח}_0 & {}_1\text{ח}_1 & {}_1\text{ח}_2 & {}_1\text{ח}_3 \\ {}_2\text{ח}_0 & {}_2\text{ח}_1 & {}_2\text{ח}_2 & {}_2\text{ח}_3 \\ {}_3\text{ח}_0 & {}_3\text{ח}_1 & {}_3\text{ח}_2 & {}_3\text{ח}_3 \end{bmatrix} = \begin{bmatrix} {}^1\text{ח}^1 & {}^1\text{ח} & {}^1\text{ח}_1 & {}^1\text{ח}_2 \\ \text{ח}^1 & {}^0_0\text{ח}^0_0 & \text{ח}_1 & \text{ח}_2 \\ {}_1\text{ח}^1 & {}_1\text{ח} & {}_1\text{ח}_1 & {}_1\text{ח}_2 \\ {}_2\text{ח}^1 & {}_2\text{ח} & {}_2\text{ח}_1 & {}_2\text{ח}_2 \end{bmatrix} \tag{2.89}$$

More conventionally, this second statement becomes the following, where we have reiterated the power function as conventionally stated on the right. It is of interest to note the orthogonal nature of the matrix by the sense of each function, and that the inertial constant is inherently an imaginary invariant, so that making its sense explicit would effectively rotate all the functions clockwise $\frac{\pi}{2}$, from their positions as follows.

$$\begin{array}{lllll} P_0 = -\text{ח}\kappa_0^{-1}\omega_0^{-1} & I_0 = -i\text{ח}\kappa_0^{-1} & \hbar = \text{ח}\kappa_0^{-1}\omega_0 & E_0 = i\text{ח}\kappa_0^{-1}\omega_0^2 & \underline{\left(P_0 = -\text{ח}\kappa_0^{-1}\omega_0^3\right)} \\ \mathrm{Y}_0 = -i\text{ח}\omega_0^{-1} & \text{ח}\left(=\frac{m_0}{i\kappa_0}\right) & G_0 = i\text{ח}\omega_0 & \tau_0 = -\text{ח}\omega_0^2 & \\ \frac{m_0}{\omega_0} = \text{ח}\kappa_0\omega_0^{-1} & m_0 = i\text{ח}\kappa_0 & Z_0 = -\text{ח}\kappa_0\omega_0 & \tau_1 = -i\text{ח}\kappa_0\omega_0^2 & \\ \mathrm{Y}_2 = i\text{ח}\kappa_0^2\omega_0^{-1} & \lambda_0 = -\text{ח}\kappa_0^2 & G_2 = -i\text{ח}\kappa_0^2\omega_0 & f_0 = \text{ח}\kappa_0^2\omega_0^2 & \end{array} \tag{2.90}$$

The additional 9 functions are:

Mechanical Impedance, Z_0 $\quad {}_1\text{ח}_1 \equiv -\text{ח}\kappa_0\omega_0$ (2.91)

Transverse Momentum Surface Density, G_2

$${}_2\text{ח}_1 \equiv -i\text{ח}\kappa_0^2\omega_0 \tag{2.92}$$

Planck's Quantum of Action, $\hbar$ $\quad {}^1\text{ח}_1 \equiv \text{ח}\kappa_0^{-1}\omega_0 = \text{ח}c_0$ (2.93)

(Spin Angular Momentum)

Linear Transverse Force Density, τ_1 $\quad {}_1\text{ח}_2 \equiv -i\text{ח}\kappa_0\omega_0^2$ (2.94)

Wave Stress, f_0 $\quad {}_2\text{ח}_2 \equiv {}^2\text{ח}^2 \equiv {}^2\text{ח}_2 \equiv {}_2\text{ח}^2 \equiv \text{ח}\kappa_0^2\omega_0^2$ (2.95)

Spin Energy, E_0 $\quad {}^1\text{ח}_2 \equiv i\text{ח}\kappa_0^{-1}\omega_0^2$ (2.96)

Mass Frequency Ratio, $\frac{m_0}{\omega_0}$ $\quad {}_1\text{ח}^1 \equiv \text{ח}\kappa_0\omega_0^{-1} = \frac{\text{ח}}{c_0}$ (2.97)

Yank Surface Density, Y_2 $\quad {}_2\text{ח}^1 \equiv i\text{ח}\kappa_0^2\omega_0^{-1}$ (2.98)

Wave Power, P_0 $\quad {}^1\text{ח}_3 \equiv {}^1\text{ח}^1 \equiv -\text{ח}\kappa_0\omega_0^3$ (2.99)

(Yank Volume Density, Y_3) $\quad {}_3\text{ח}_3 \equiv -\text{ח}\kappa_0^3\omega_0^3$ (2.100)

These derivative functions for a fundamental rotational oscillation, the neutron, are invariant functions of the resonant frequency, ω_0, and wave number, κ_0, of the continuum and not of any linear dimension of space or time. These latter two time and space parameters serve to gauge the interaction of the various functions and in fact to set the gauge, which is also the basis for the metric, for space and time itself. We can show this in greater detail in the orthogonal matrix that follows. The functions are instantaneous vectors, which together form a rotational tensor or spinor. The functions with asterisks, all of which are of real sense, are primary invariants of the system. Thus, while all are invariant with respect to the fundamental oscillation, with beta decay, for example, *E* and *m* for the central oscillation changes, though not for the system as a whole, which is conservative. The primary invariants with multiple asterisks can be more readily seen as properties of the continuum itself, with the single asterisks indicating invariants of oscillation, but all are rightly seen as spin potentials of the continuum. The primacy of the multiples is seen in (2.68) which is an expression of these four describing the necessary conditions for wave motion. Thus

$$\lambda_0 \equiv \daleth\kappa_0^2 = \frac{1}{\frac{\omega_0^2}{\kappa_0^2}}\daleth\omega_0^2 \equiv \frac{1}{\frac{\omega_0^2}{\kappa_0^2}}\tau_0 = \frac{f_0}{\omega_0^2}, \tag{2.101}$$

while generally unrecognized is

$$\daleth \equiv \frac{1}{\kappa_0^2\omega_0^2} f_0 \tag{2.102}$$

or diagrammatically

$$\begin{array}{ccc} \daleth & \overset{\omega_0^2}{\longleftrightarrow} & \tau_0 \\ \updownarrow \kappa_0^2 & \kappa_0^2\omega_0^2 \nwarrow\!\searrow\!\swarrow\!\nearrow \frac{\omega_0^2}{\kappa_0^2} & \updownarrow \kappa_0^2 \\ \lambda_0 & \overset{\omega_0^2}{\longleftrightarrow} & f_0 \end{array}. \tag{2.103}$$

A similar condition with respect to the action, impedance, power and mass/frequency ratio is

$$\begin{array}{ccc} \hbar & \overset{\omega_0^2}{\longleftrightarrow} & P_0 \\ \updownarrow \kappa_0^2 & \kappa_0^2\omega_0^2 \nwarrow\!\searrow\!\swarrow\!\nearrow \frac{\omega_0^2}{\kappa_0^2} & \updownarrow \kappa_0^2 \\ Z_0 & \overset{\omega_0^2}{\longleftrightarrow} & \frac{\daleth}{c_0} \end{array} \tag{2.104}$$

Among the secondary invariants, those whose magnitudes for a given particle vary from the fundamental, but whose relationships are still gauged as with the primary invariants, we have one involving the well known equation of Einstein,

$$\begin{array}{ccc} I_0 & \overset{\omega_0^2}{\longleftrightarrow} & E_0 \\ \updownarrow \kappa_0^2 & \kappa_0^2\omega_0^2 \nwarrow\!\searrow\!\swarrow\!\nearrow \frac{\omega_0^2}{\kappa_0^2} & \updownarrow \kappa_0^2 \\ m_0 & \overset{\omega_0^2}{\longleftrightarrow} & \tau_1 \end{array} \tag{2.105}$$

In fact, the coupling between τ_0 and f_0 gauges the gravitational interaction, as the quantum of gravity is given

$$\tau_0 = -\mathrm{A}_0 f_0 = -\mathrm{A}_0 \frac{T_0}{6\sqrt{3}} = T_0 \frac{1}{6\sqrt{3}\left(-i\kappa\right)_0^2} \tag{2.106}$$

$$G_q = \frac{d\tau_0}{dT_0} = -\frac{1}{6\sqrt{3}\kappa_0^2} \tag{2.107}$$

where, by virtue of the spin mechanics of $\left(\frac{V}{C}\right)$, the isotropic 4-stress, T_0, operates on one pair of diagonal h-axis, at an angle of 0.615479…, relative to the six cubic faces (and normal to any three ortho-normal co-ordinates of a 3-space,) so that

$$T_0 = 6\sqrt{3} f_0 = 6\sqrt{3}\tau_0 \mathrm{A}_0^{-1} \text{ and} \tag{2.108}$$

$$\mathrm{A}_0 = 6\sqrt{3}\tau_0 T_0^{-1}. \tag{2.109}$$

We can state the relationship between the spin functions and that 4-stress as

$$_{\kappa}\sqcap_{\omega} = \sqcap_{\mu\nu} = 4\pi\left(2T_{\mu\nu}\right) = 8\pi T_{\mu\nu} \tag{2.110}$$

Here $2\,T_{\mu\nu}$ is the 4-stress correspondence of the 3-strain component in (2.59)-(2.62), and 4π integrates the surface stress oscillation over a spherical surface over one cycle. Since $\sqcap_{\mu\nu}$ is a geometrically defined set of functions as developed above, this is a background-independent quantum solution to the field equations of general relativity.

The following orthogonal matrix is the multi-faceted jewel of this rotational oscillatory system. In addition to the above functions, there are two others shown in the bottom two spots of the next to the rightmost column. *Y*, not to be confused with yank, Y, is the tension or Young's modulus of elasticity of the continuum, while the function under it is the inverse of the Planck area. Notice that it is gauged by the same general derivative, κ_0^2, with the stress f_0, as the stress is with the transverse, and in the case of the rotational oscillation, central wave force, τ_0. Thus the Planck area, from (2.109), is a derivative of area with respect to f_0, or

$$\mathrm{A}_{Planck} = \frac{d\mathrm{A}_0}{dT_0} = -\frac{6\sqrt{3}\tau_0}{\left(6\sqrt{3}\right)^2 f_0^2} = \frac{\sqcap\omega_0^2}{6\sqrt{3}\sqcap^2\kappa_0^4\omega_0^4} = \frac{\sqcap}{6\sqrt{3}\sqcap^2\kappa_0^6 c^2} = -\frac{G_q}{\lambda_0^2}\frac{\sqcap}{c^2} = -G_N\frac{\hbar}{c^3} \tag{2.111}$$

Of similar interest is the relationship of the mechanical impedance with respect to Planck's quantum as

$$\hbar = 6\sqrt{3} Z_0 G_q. \tag{2.112}$$

Finally, the functions $\hbar$ and G are gauged by κ_0, where G_0 is related to the quantum of charge, *e*, by

$$G_0 = i\kappa_0\hbar = i\sqcap\omega_0 = \frac{\pi}{1.002406...}e. \tag{2.113}$$

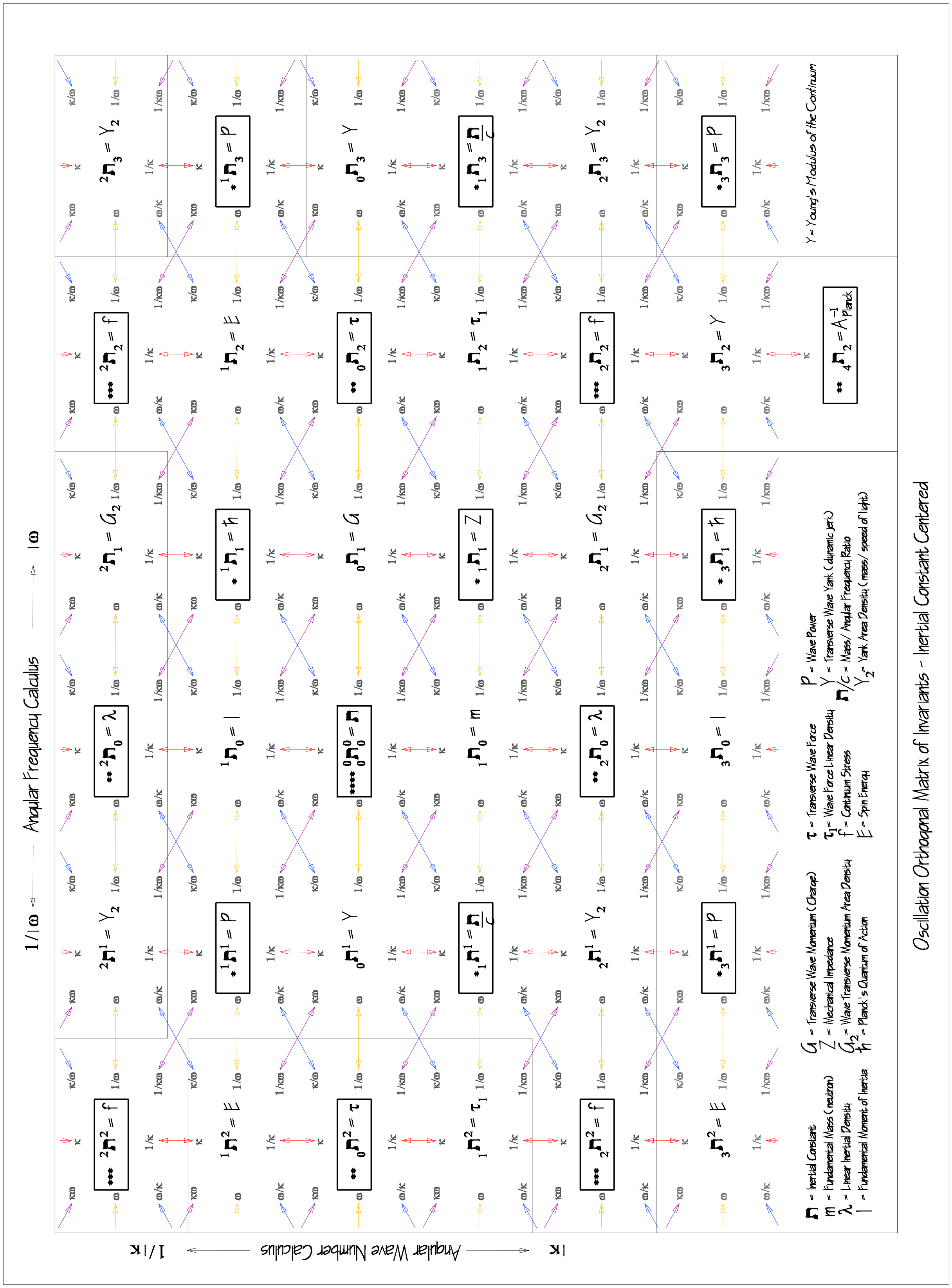
thus

$$\frac{1.002406...}{\pi}\kappa_0 = -i\frac{e}{\hbar} = -i\frac{e}{\sqcap c}. \tag{2.114}$$

The ratio of the gravitational quantum and the charge quantum, then is

$$\begin{aligned} \frac{e}{G_q} &= i\kappa_0 Z_0 \frac{6\sqrt{3}\left(1.002406...\right)}{\pi} \\ &= i\sqcap\kappa_0^2\omega_0 \frac{6\sqrt{3}\left(1.002406...\right)}{\pi} \\ &= G_2 \frac{6\sqrt{3}\left(1.002406...\right)}{\pi} \end{aligned} \tag{2.115}$$

The invariant functions, then, are seen to be invariant differentials and the various instances of $\pm i\kappa$ and $\pm i\omega$, are their co-variant derivatives. Related to this is the fact



Matrix of Invariants

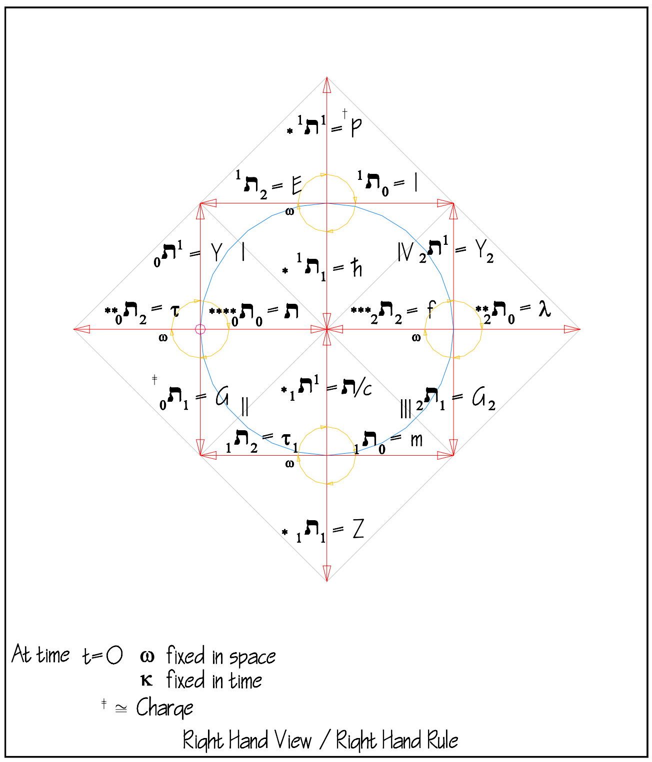
that, as indicated by the prefix noughts, the fundamental values of these functions constitute a system of natural dimensional unit values. Thus we have the following SI equivalents for the derivatives, imaginary sense omitted, along with the inertial constant, where θ is understood to be 1, which are the fundamental units of space and time, though they are obviously not quantum in the conventional sense of being discrete and indivisible. They represent, instead, the classical foundational parameters of the quantum world.

$$\sqcap = 3.51767\ldots x10^{-43}\; kg \cdot m/\theta \tag{2.116}$$

$$x_0 = \kappa_0^{-1} = 2.10019\ldots x10^{-16}\; m/\theta \tag{2.117}$$

$$t_0 = \omega_0^{-1} = 7.00549\ldots x10^{-25}\; s/\theta \tag{2.118}$$

This matrix can also be represented to advantage by the following spin charts. The first, labeled, Right Hand View, Right Hand Rule, can be thought of as being prior to a strain and hence oscillation and is marked as at time t = 0. All time functions of $\sqcap\omega_0^n$ are fixed in the same direction and therefore in phase and all space functions of $\sqcap\kappa_0^n$ are arrayed together in sequential order with a counterclockwise rotation of the cycle starting at the left or 9:00. This represents a spin potential, analogous in some respects to a vector potential.



Spin Chart 1 – t = 0, RHV/RHR

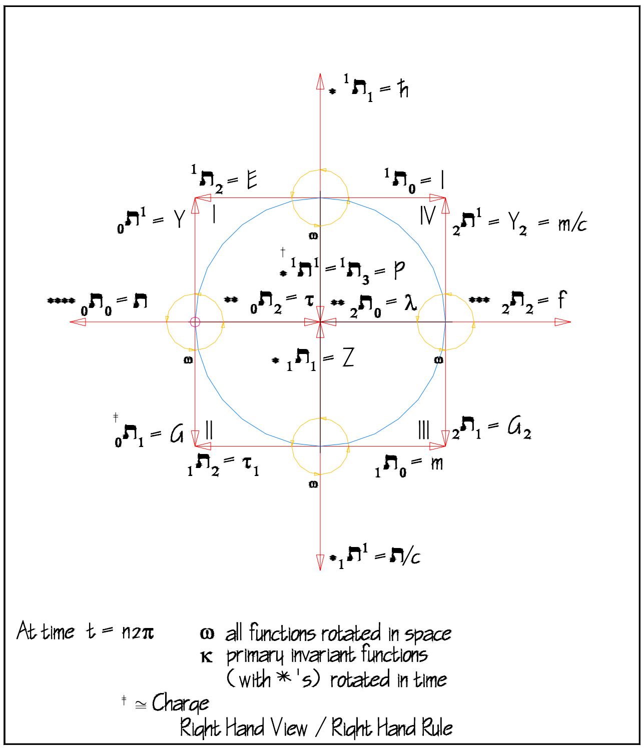
We can imagine an X cut along the 45 degree lines through the center of the circle which is lying in an x-y plane. (Assume the top of the page is initially in the $+x$ direction for reasons that will become clear.)

1. Pull the bottom arrow head of ת/c, which is pointing up toward +x at the center point, toward the –z direction, down out of the page, and pull the top arrow head of $\hbar$, which is pointing down at the center point, toward +z, so that the circle is now in the y-z plane, facing in the –x direction.
    a. Now, the vectors $\hbar$ will be pointing up and P down, Z up and ת/c down (and facing away from you) all having undergone a +i or $\pi/2$ ccw directional rotation about their group source or origin (viewed toward the center from the +y direction).
    b. This transformation performs an axial +i rotation of the remaining vectors of their groups, E and I for ^{1}ת and τ_1 and m for $_{1}$ת, along with a simultaneous cw orbital rotation about the y axis.
    c. This also produces a -i axial rotation of τ-ת of $_{0}$ת and f-λ of $_{2}$ת and a simultaneous directional -i rotation of Y-G and Y_2-G_2 about the y axis.

   As a result, all the vectors will have undergone either an axial, axial/orbital or a directional rotation.
2. Simultaneous with (1), but second in a non-commutative order, in group $_{0}$ת pull the arrow head of ת which is pointing right at the center point, toward the -z direction (down out of the page) and in group $_{2}$ת on the right pull the arrow head of f, which is pointing left at the center point, toward the +z direction. The tendency or differential effect of this transformation, which is similar to (1), is to place the circle in the x-z plane, but this is in conflict with (1) which puts the circle in y-z. Since $\hbar$ is the spin angular momentum vector, we will give it a differential precedence, i.e. first in a non-commutative order, but after the full $\pi/2$ directional rotation of (1) and of $_{0}$ת and $_{2}$ת about Y-G and Y_2-G_2, we end up with
    a. The transformation of (1.a-c) rotated –i about the z axis (viewed once again toward the origin from the +z direction) into the x-z plane.
    b. Now, ת points outward toward +x and τ inward toward -x, and λ points inward toward +x and f points outward toward -x.
    c. This produces a +i axial rotation of Y-G and Y_2-G_2 along with a –i orbital rotation about the $z = \hbar$ - ת/c axis.
3. The result is that all vectors of the groups undergo both a directional and an axial rotation, though the differential precedence or sequence varies. For the primary invariants, denoted by asterisks, of groups ^{1}ת and $_{1}$ת the non-commutative sequence is (1)directional-(2)axial, while for groups $_{0}$ת and $_{2}$ת the sequence is (1)axial-(2)directional. For the remaining components which meet at the corners of the square, there is also an orbital rotation along with the axial. The sequence is the same as for the primary invariants of their group. The result is Spin Chart 2, the diagram of time $\boldsymbol{t = n2\pi}$ which is in the x-z plane. Note that the primary invariants are all rotated in time with respect to diagram $\boldsymbol{t = 0}$.

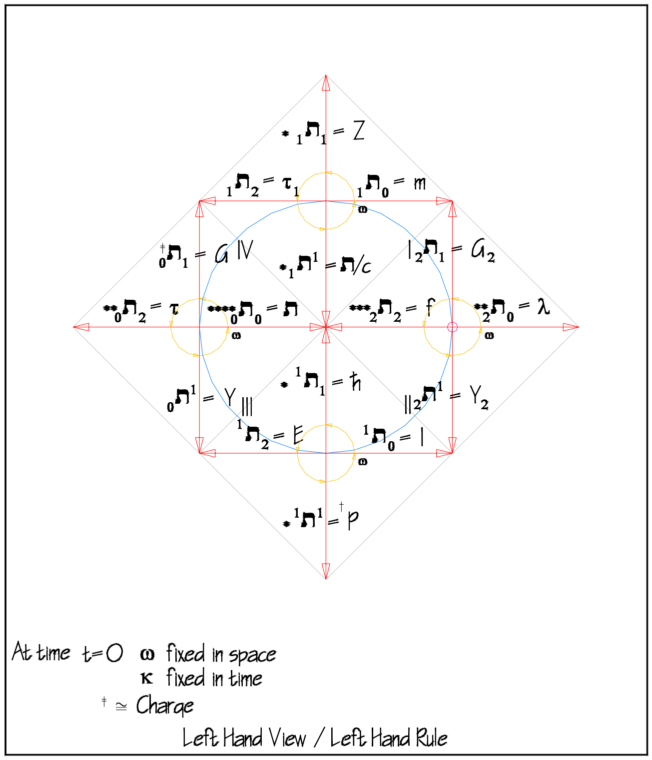
With an actual kinetic model, in the initial condition, the **RHV/RHR** label on each group square will be facing out, all right side up. In the final condition the model would be viewed on edge, with the right hand labels all facing to the right, but with the top and bottom square labels inverted and the left hand labels facing left, and similarly oriented.

To complete the symmetry of the picture we can also show a Left Hand View, Left Hand Rule in Spin Chart 3 that would be applied to the back of the **RHV/RHR** diagram. It can readily be seen that if you place the **LHV/LHR**, centered on the back of **RHV/RHR** and invert it 180 degrees, all the functions will line up back to back with each other. However, as we have stipulated that Spin Chart 3 follows the left hand rule, we will not invert the diagram, so that after the above operations are performed, the spin vectors are correct, so that $\hbar_{RHR} = -\hbar_{LHR}$ and their physical rotation representation is the same, i.e. $\hbar_{RHR}$ is ccw and $\hbar_{LHR}$ is cw when viewed from the vector arrow toward the center of the configuration.

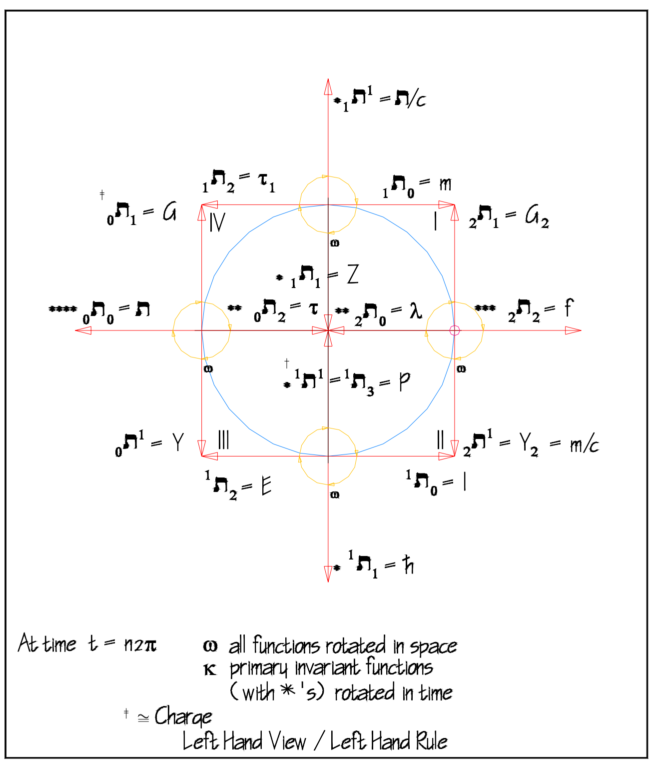


Spin Chart 2 – t = n2π, RHV/RHR

The result of this juxtaposition shows that Spin Chart 2 corresponds with the condition in Spin Diagram 1, at the +*y* position at the cross of the +*y* path. Spin Chart 4 corresponds with the similar position on the –*y* path. Both charts are viewed from the direction toward which θ is rotating at the +*y* and –*y* crossing. This corresponds with the condition found at Figure 2, in the section on EM waves.



Spin Chart 3 – t = 0, LHV/LHR



Spin Chart 4 – t = n2π, LHV/LHR

Quantum Rotational Oscillation or Spin

We can create a simple physical model of this condition, albeit in 2-D mode, using a flexible drumhead or a planar frame such as a crochet hoop over which we stretch a thin, flexible membrane. Pierce the center with a machine bolt with washers, head up and tightened so that it cannot slip in the opening, and preferably of greater length than the radius of the hoop. Fold the length of the bolt, representing the *X* axis, up against the bottom of the membrane or hoop and rotate the distal end around the hoop's circumference, allowing the bolt to slip and rotate freely in your loose grip. In the following, the lower case letters, *x*, *y* and *z*, refer to physical points originally in the undisturbed reference plane but now subject to displacement or strain and the upper case to the physical reference frame, *X*, *Y* and *Z* or to a functional point on the path of the strain, *K*, *V*, *M*, *E*, and *W*. In these graphics, the *X*, *Y* and *Z* axes have been rotated $+i$ from their customary positions, which amounts to a $\frac{2\pi}{3}$ ccw rotation about the *XYZ* diagonal. Thus $+V_{-z}$ indicates point $-z$ displaced to functional point $+V$. $+K_{-z}$ indicates $-z$ at *K* moving in the positive direction, toward $+V$. We will reference Wave Diagram 1 in this discussion.

The disk, ϕ, (+, originally up) and (–, originally down), formed by the bolt head and washer will now be roughly perpendicular to its original position in the plane of the membrane and hoop, θ, (+, up) and (–, down). The bottom edge of the disk, which we will call point $-z$, will be depressed to a point we will call $-V_{-z}$, while the point across the diameter at $+z$ will be elevated to $+V_{+z}$. The point $-y$ clockwise $\frac{\pi}{2}$ from $-V_{-z}$ on the edge of the disk and in the plane of the membrane, we will designate at $-iK_{-z} = K_{-y}$, and the diametrically opposed point $+y$ on the disk we will call at $+iK_{-z} = -iK_{+z}$.

As we move the bolt distal end $+i\theta_+$ (counterclockwise ½ π) around the hoop, the bolt head and adjacent membrane will rotate $-i\phi_+$ (clockwise ½ π)), as would be indicated by an axial vector pointing with the bolt along the bottom side of the distorted membrane. The sense subscripts indicate the side from which the rotation is observed. Point $-z$ will move up in the direction of the plane of the hoop, $-i\phi_+$, but $+i\theta_+$ to point $+K_{-z}$ in the plane of the hoop. Simultaneously, point $+z$ moves down toward the hoop plane, once again $-i\phi_+$, with respect to the disk and $+i\theta_+$ with respect to the hoop to point $-K_{+z}$. Another ccw ½ π rotation of the bolt around the hoop carries $-z$ up to $+V$ and $+z$ down to $-V$, once again with cw motion around the disk and ccw motion with respect to the hoop and $+V$. A third ccw ½ π rotation of the bolt carries $-z$ down to $-K_{-z}$ and +z up to $+K_{+z}$ in the *Y*-*Z* plane. A fourth such rotation returns carries $-z$ and $+z$ to their initial displaced positions along the *X* axis. Note that both θ and ϕ undergo a full rotation, yet continuity is maintained with no twisting up of the medium, in fact, it occurs because the medium will not allow such indefinite distortion. The matrix in (2.61) describes this condition.

Note that this motion brings together functional point $-iK_{-z}$ and physical point $-z$ from functional point $-V_{-z}$ to a rest or undisturbed point $-Z = +K_{-z}$, simultaneously with $+z$ to $+Z$ and $+V_{+z}$ and $-iK_{+z}$ together at $-K_{+z}$. It also indicates the following conceptual spatial and time identities

$$-Y \equiv \left[K_{-y}\left(x\right) = -i_\theta K_{-z} = +i_\theta K_{+z} \right] \equiv \left[K_{-y}\left(t\right) = +K_{-y}, -K_{-y} \right] \qquad (2.119)$$

and

$$+Y \equiv \left[K_{+y}\left(x\right) = -i_{\theta}K_{+z} = +i_{\theta}K_{-z} \right] \equiv \left[K_{+y}\left(t\right) = -K_{+y}, +K_{+y} \right]. \qquad (2.120)$$

Clearly, $-V_{\phi}$ and $+V_{\phi}$ are two stable or observationally <u>static</u> <u>functional</u> points comprised of the constant <u>physical</u> <u>flux</u> of ϕ over one rotation of θ, while $-K_{\phi}$ and $+K_{\phi}$ form two iterations of a <u>moving</u> <u>functional</u> K_{θ} path around the $-V_{\phi}$ – $+V_{\phi}$ axis in the θ plane comprised of the successive points of ϕ while at momentary <u>physical</u> <u>rest</u>. Thus, each point of ϕ coincides with a unique V once and a unique K twice with each cycle of θ. In terms of simple harmonic motion, the potential energy function of V, which we might surmise is also the energy of quantum static electrical charge or the electric field, is comprised of the continuum's sustained displacement-strain created by its oscillatory motion, while the kinetic energy function of K, the quantum manifestation of the energy of a magnetically induced current, is comprised of the sustained maximum momentum created by continuous change in rest position.

This is in keeping with Wave Diagram 2 in which the position of zero displacement corresponds with the instance of greatest momentum and zero acceleration and force. It is also what (2.101) would appear to indicate, since λ_0 is the inertial density, a function of position, $\phi''(x)$, and τ_0 is the tension force, a function of time, $\phi''(t)$, of the spacetime continuum, as

$${}_{2}\text{ת}_{2} \equiv \lambda_0\omega^2 \equiv -\left(\text{ת}\kappa_0^2\right)\omega^2 = -\left(\text{ת}\omega_0^2\right)\kappa^2 \equiv \tau_0\kappa^2 \equiv {}_{2}\text{ת}_{2} \qquad (2.121)$$

.

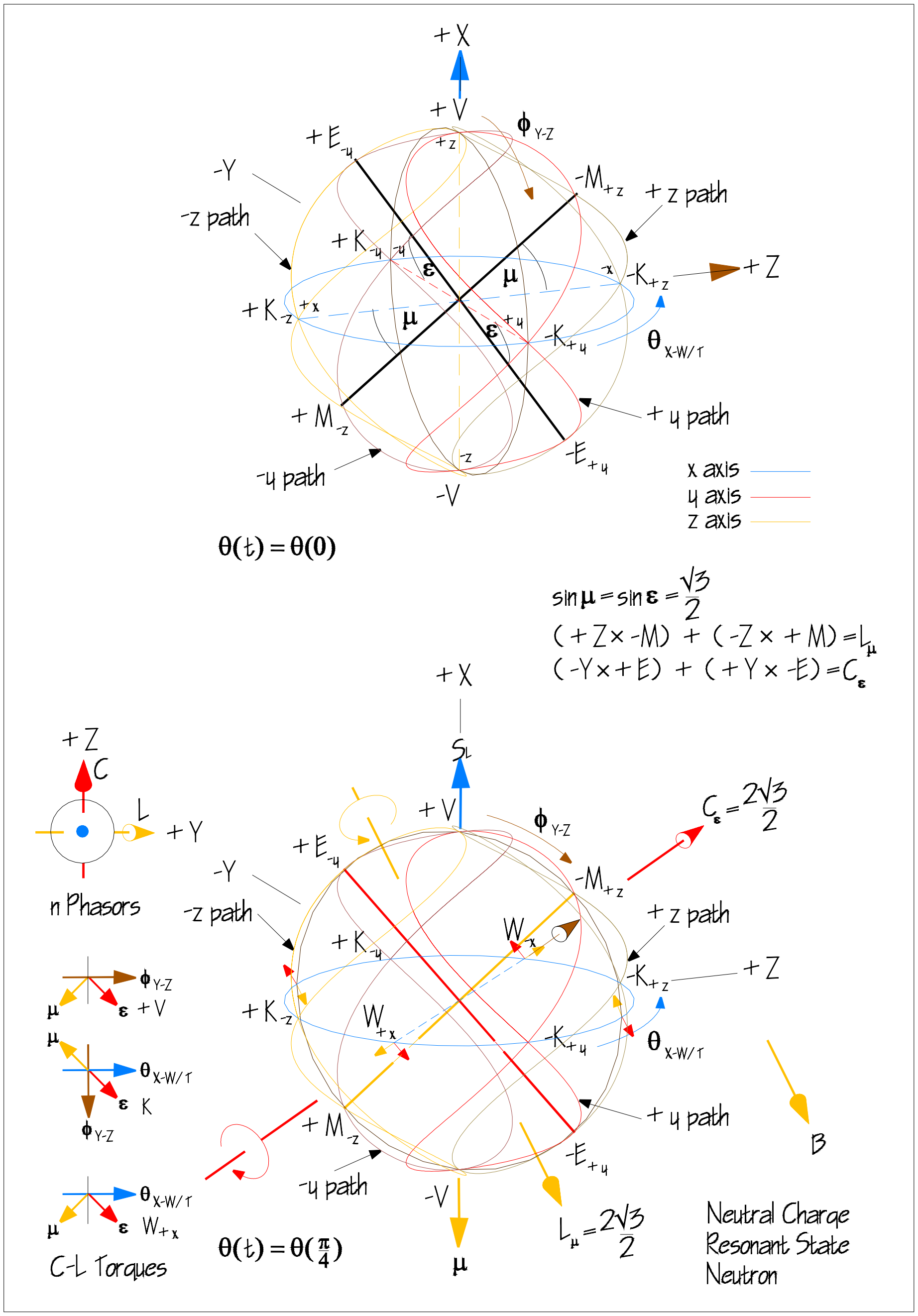
Wave Diagram 1 assumes a point $\omega t_1 < ¼\ \pi$. At $t_0 = 0$, +z was aligned with the X axis, and ϕ was aligned with the X-Y plane, normal to the plane of the paper. Rotation of θ brings $+x$ and the y-z orthogonal axes to the positions shown by the small red disks and radial lines within ϕ. The bold **X** superimposed on ϕ is the congregation of the $-M$, $-E$, $+M$, and $+E$ functional lines shown in Wave Diagram 2, and is the analogous condition for the quantum oscillation, in this case all being cotemporaneous in space as shown and rotating in time around the X axis. As will be shown in the spin diagrams that follow, the cross product of these vectors and their initial position in the Y-Z plane, operating through the stress-strain function, results in the creation of a wave guide at the points W_{+x} and W_{-x} (the latter not shown in the diagram) which is the boundary of the permanently rotated axis of wave travel. Such wave guide is seen to be earlier as the operation of $\left(\frac{V}{C}\right)$.

At the moment of the diagram, t_1, the ends of the **X** on the circumference of ϕ represent the displacement of points $+z'$, $+y'$, $-z'$, and $-y'$, each $\phi(¼\ \pi - t_1)$ less than $+z$, $+y$, $-z$, and $-y$. At time $t_{(¼\ \pi)}$, these last four points will have rotated that amount in ϕ and points $+x$ and $-x$ and the ends of the power cross, **X**, will have rotated a corresponding amount in θ to the points $-M$, $-E$, $+M$, and $+E$ shown. As the wave speed for θ and ϕ is identical over the short term for a free quantum waveform, i.e. the neutron, and indefinitely under certain inertial conditions of atomic nuclear congregation, $+x$ and $-x$ constitute an effective boundary of the wave. Implicit in this model is that heat and friction is wholly a transfer of kinetic energy and momentum among separate quantum waveforms through translational, i.e. Brownian, motion and is not an operant condition of individual wave dynamics, thus there is no damping involved except as a boundary condition due to stress and strain at $+x$ and $-x$ as it might effect the fundamental eigenvalues, κ_0 and ω_0.

Spin Diagram 1 shows another view of this condition, this time with the path integral of an arbitrary point $+y$ over time, where the arbitrary y-z axes are indicated by the red and yellow broken lines at time, $t = 0$. Instead of the contemporaneous M-E power axes shown in Wave Diagram 1, however, we have the same positions in sequential order for $+y$, which is at rest at the functional point K at $t = 0$ at the instant of the diagram as indicated by the rotation arrows for θ and ϕ. As θ rotates $+2\pi$ as indicated at $+X$, and ϕ, -2π at $+x$, (instantaneously $+i$ at $+Z$ in the diagram), $+y$ will etch a figure 8 path, passing in sequence through 8 equal time segments of ¼ π, through functional points –E, –V, +M, +K, +E, +V, –M, and back to –K. A continuum of other radials around the circumference of θ will do likewise, each $\partial\phi/\partial\theta$ from the adjacent ones. Four such paths of two orthogonal axes, of which $+y$ is one component, are shown.

The path of each radial is broken into four ½ π phases, two capacitive and two inductive as shown, the capacitive phase always on the leading side in the direction of rotation of θ, and the inductive side always on the lagging side counter to the direction of rotation. Each such phase can further be seen as dominated by the kinetic energy of rotational oscillation between K and E for the capacitive and between K and M for the inductive phases and by the potential energy of displacement/strain between E and V and between M and V. The sine of the torsion strain angles, ε and μ, between K-0-E and K-0-M is $\frac{\sqrt{3}}{2}$ with a cosine of ½, indicating an angle of 1/3 π, while the sine and cosine of the angle between each of the power points and the plane of θ at the center is $\frac{1}{\sqrt{2}}$, indicating an angle of ¼ π. The sine and cosine of the angle between each plane of K-0-E and K-0-M and the plane of θ at the midpoint of the radial 0-K is $\frac{\sqrt{2}}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}$, indicating an angle of .9553166, and an angle between the planes of angles ε and μ and the X axis of .6154797.

It bears emphasizing that the cross products, $K \times E$ and $K \times M$, used in the following development and the corresponding diagrams represent the wave strain and are between the rest points with zero angular wave strain, K, and the points of strain at maximum instantaneous power, M and E. With respect to Spin Diagrams 1 and 2, this strain is balanced in resonance between the capacitive and the inductive phases. With the remaining diagrams, which involve a rotation of ϕ within θ and the Y-Z plane in the case of Diagrams 3 and 5 and an inversion of one axis in the case of Diagrams 4 and 6, one or the other of the phases predominates, indicating a sustained capacitive or inductive state of the wave. We would expect the predominant strain due to cosmic expansion to be a sustained inductive state or conversion of potential to kinetic energy, just as we might expect a capacitive state in association with a sustained condition of overall or local cosmic contraction.



Spin Diagram 2 – Neutron

Spin Diagram 2, top figure, shows the oscillation at a point in time, *t(0)* = -*K*, for +*y*, shown as -K_{+y} with the instantaneous power moments of -*M*-+*M* and -*E* -+*E*. At time *t(0)*, these moments, shown by the dark cross, are not the active moments, the latter of which would be, as always, in the ϕ plane, if they were shown. Those shown are instead the positions to be reached by the strain/displacement of the *y-z* axes when they reach those physical points and assume those functional positions. The bottom figure shows this momentary assumption. It represents a point at $t(\frac{1}{4}\pi)$ where $+y$ and $+z$ have rotated from the readers perspective $-\frac{1}{4}\pi$ to $-E_{+y}$ and $-M_{+z}$ respectively in ϕ. Boundary nodes W_{+x} and W_{-x} and therefore ϕ have rotated $+\frac{1}{4}\pi$ in θ. In the top figure, the tangential velocity, $\eta'(t)$, and thus momentum is at a maximum at the two poles of $-K_{+y}$ and $+K_{-y}$, to be followed at $t(\frac{1}{2}\pi)$ by the maximum for $\zeta'(t)$ at $-K_{+z}$ and $+K_{-z}$. The maxima, then, rotate with ϕ about θ, $\frac{1}{2}\pi$ out of phase from the W's. The rotation of θ creates a spin angular momentum vector, S_L, shown at $+X$. The power moments rotate with θ, at the same angular frequency and at the same phase position. Due to the steady state dynamics of the quantum oscillation, we can assign a particular dynamic component, i.e. force or momentum in equal measure or one quarter of the total to each of the power moments, as in the following.

The torsional strains, ε and μ, each crossed from the rest position to the extremum, create a torque on each side of the oscillation which is shown by the axial vectors C_ε and L_μ and which can be described as

$$\left(+\mathbf{Z}\times\text{-}\mathbf{M}\right)+\left(-\mathbf{Z}\times+\mathbf{M}\right)=\mathbf{L}_{\mu} \tag{2.122}$$

$$\left[\left(+\kappa_0^{-1}\times\tfrac{-1}{4}\text{ת}\omega_0^2\right)+\left(-\kappa_0^{-1}\times\tfrac{1}{4}\text{ת}\omega_0^2\right)\right]\mathbf{sin}\,\mu=i\tfrac{\sqrt{3}}{2}\kappa_0^{-1}\left(\tfrac{1}{2}\text{ת}\omega_0^2\right)=i\tfrac{\sqrt{3}}{4}\hbar\omega_0 \tag{2.123}$$

$$\left(+\mathbf{Y}\times\text{-}\mathbf{E}\right)+\left(-\mathbf{Y}\times+\mathbf{E}\right)=\mathbf{C}_{\varepsilon} \tag{2.124}$$

$$\left[\left(+\kappa_0^{-1}\times\tfrac{-1}{4}\text{ת}\omega_0^2\right)+\left(-\kappa_0^{-1}\times\tfrac{1}{4}\text{ת}\omega_0^2\right)\right]\mathbf{sin}\,\varepsilon=i\tfrac{\sqrt{3}}{2}\kappa_0^{-1}\left(\tfrac{1}{2}\text{ת}\omega_0^2\right)=i\tfrac{\sqrt{3}}{4}\hbar\omega_0 \tag{2.125}$$

The i's, as orthogonal sense, arise naturally in the crossing process, as in $j\times k=i$. These torques rotate in concert with the torsional strains and with θ. The torques are represented by the (active) h-axes detailed above and travel at the same angular velocity as θ and ϕ. It bears noting that while both C_ε and L_μ are shown according to the right hand rule, the force embodied in the strain at M is decreasing, while the momentum is increasing. At E we have the opposite case, where the momentum is decreasing while the force is increasing. The rotational sense of both vectors is in the direction of increasing force, forward in time for E and backward in time for M. From the point of view of motion, however, the rotation at L_μ is clockwise and would be represented by a left hand axial vector. This is in keeping with the left hand rule for induction. If we assume that L_μ is the direction of the magnetic field and cross it into the flow path between M and K on the spherical surface, the product, a force vector, will be in the direction of the oscillatory center.

The sum of (2.123) and (2.125) is a complex function of ϕ, in which we find the dimensions of spin energy, E_0, and in fact it is the spin energy of the system, which with complex integration with respect to a quarter cycle of time gives us the spin angular momentum as

$$S_L=i\tfrac{\sqrt{3}}{2}\text{ת}\omega_0^2\kappa_0^{-1}\int_{\omega t=0}^{\frac{\pi}{2}}e^{i\theta}=\tfrac{\sqrt{3}}{2}\int_t {}^1\text{ת}_2=\tfrac{\sqrt{3}}{2}\,{}^1\text{ת}_1=\tfrac{\sqrt{3}}{2}\hbar\,. \tag{2.126}$$

Had we started with the momenta of the moments being used, instead of the force in (2.122) and (2.124), we would have arrived at the action directly since dotting the momenta into the inverse wave number effectively integrates the momenta. Thus, instead of (2.123) and (2.125) we would have

$$\begin{aligned} S_{L\mu} &= \left[\left(\tfrac{-1}{4} \sqcap \omega_0 \cdot + \kappa_0^{-1} \right) + \left(\tfrac{1}{4} \sqcap \omega_0^2 \cdot - \kappa_0^{-1} \right) \right] \mathbf{cos} \left(\tfrac{\pi}{2} - \mu \right) \\ &= -\tfrac{\sqrt{3}}{2} \kappa_0^{-1} \left(\tfrac{1}{2} \sqcap \omega_0 \right) = -\tfrac{\sqrt{3}}{4} \hbar \end{aligned} \tag{2.127}$$

$$\begin{aligned} S_{L\varepsilon} &= \left[\left(\tfrac{-1}{4} \sqcap \omega_0 \cdot + \kappa_0^{-1} \right) + \left(\tfrac{1}{4} \sqcap \omega_0^2 \cdot - \kappa_0^{-1} \right) \right] \mathbf{cos} \left(\tfrac{\pi}{2} - \varepsilon \right) \\ &= -\tfrac{\sqrt{3}}{2} \kappa_0^{-1} \left(\tfrac{1}{2} \sqcap \omega_0 \right) = -\tfrac{\sqrt{3}}{4} \hbar \end{aligned} \tag{2.128}$$

$$S_L = S_{L\mu} + S_{L\varepsilon} = -\tfrac{\sqrt{3}}{2} \hbar \tag{2.129}$$

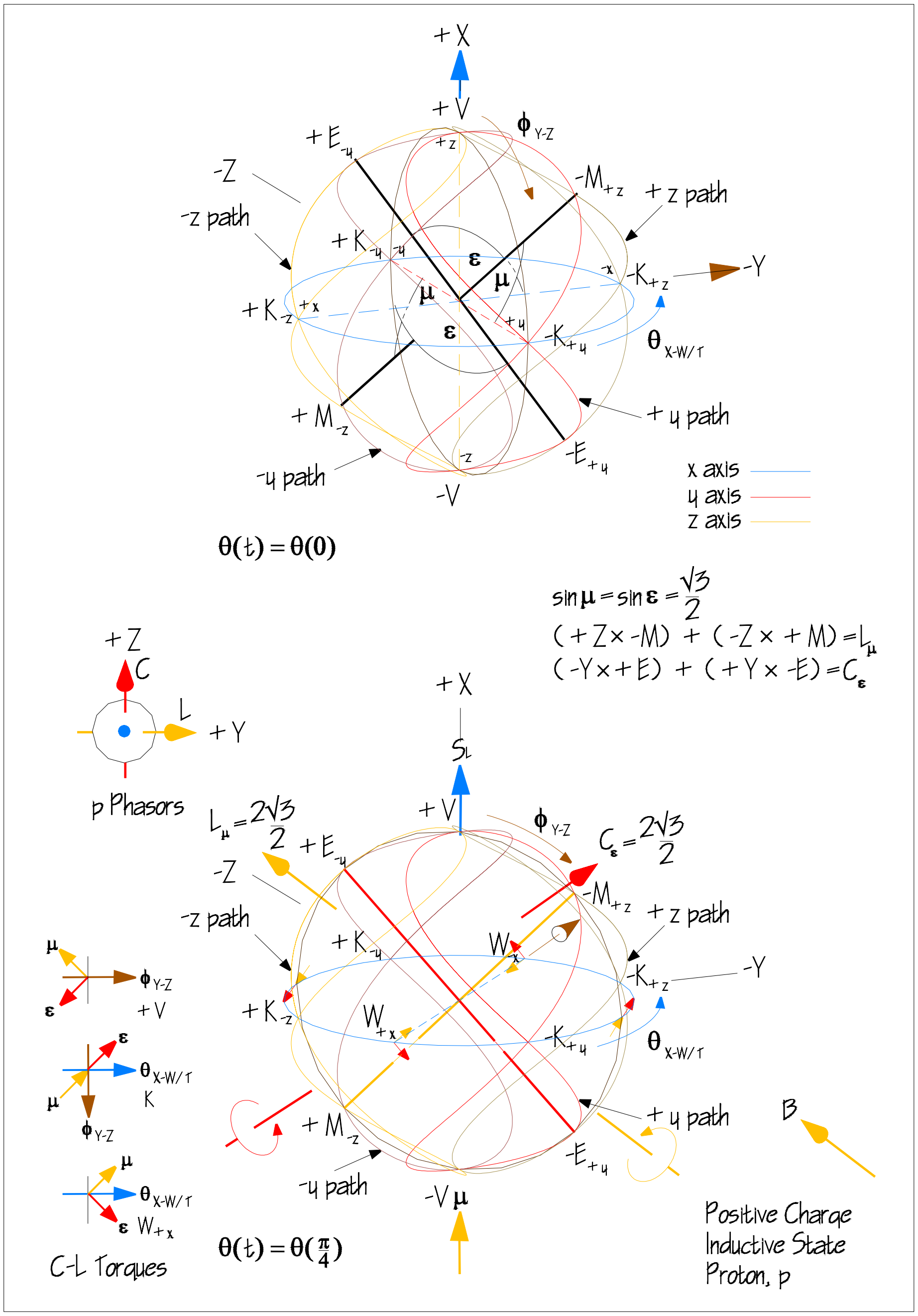
Vector μ is the effective magnetic moment vector of the oscillation which is the time averaged direction of L_μ. Vector B is the generalized direction of an assumed magnetic field. Were B to be actualized, L_μ would align with B, and S_L and μ would precess around B in the opposite direction to θ, which is in the direction indicated by L_μ. As shown in the phasor chart to the left of the diagram, in which the spin vector points toward the reader and the $+X$ direction, C leads L by ½ π.

Torques C_ε and L_μ in turn exert an equal shearing strain as shown by the small arrows on the ϕ boundary nodes W_{+x} and W_{-x}, and at $+V$ and $–V$ and again at the intersections of ϕ and θ, which we will call K_0 (between $-K_{+y}$ and $-K_{+z}$ in the top drawing) and K_π. The three charts labeled “C-L Torques” show the condition at $+V$, K_0 and W_{+x}, all as viewed from the exterior of the oscillation. The three antipodes of these, $-V$, K_π and W_{-x}, would be mirrored along the horizontal (with respect to the page) rotation vector of each chart, the rotation vectors being tangent differential components of the corresponding axial vectors. The six nodes/antinodes create a wave guide and boundary that works against the recoil of the strain from the x axis to the X axis.

The cross product of the small shear arrows, therefore, corresponding to L_μ and C_ε as indicated at W_{+x} and W_{-x}, form a charge vector (not shown) of relative magnitude *2/3* at each W, which is aligned with the ϕ rotation vector shown, its direction determined as indicated in the following section on charge generation. At the intersection of θ and ϕ the shear vectors cancel and the charge vanishes. In the case of resonance, as shown in Spin Diagram 2 for the neutral state or that of the neutron, the capacitive and inductive phases are balanced and there is no net charge force on the boundary nodes.

In an inductive state, that is, in a mode of discharge and current generation, the induction torque is predominant. This corresponds with a phase advance culminating in a $+i$ rotation of θ in the Y-Z plane, a long term physical increase in the rotational strain in the direction of spin. It is a release of kinetic energy over and above the normal oscillatory kinetic energy. This would be indicated by crossing the induction shear arrows into the capacitive shears at W_{+x} and W_{-x}, which in the case of Spin Diagram 2 would be anti-parallel to the ϕ rotation vector, and which would indicate a positive charge vector at the

W_{+x} boundary. With respect to L_μ as a left hand vector with respect to motion, the direction of motion of the μ shear vector will be reversed, which creates an unstable condition at W_{+x} and W_{-x} over time.



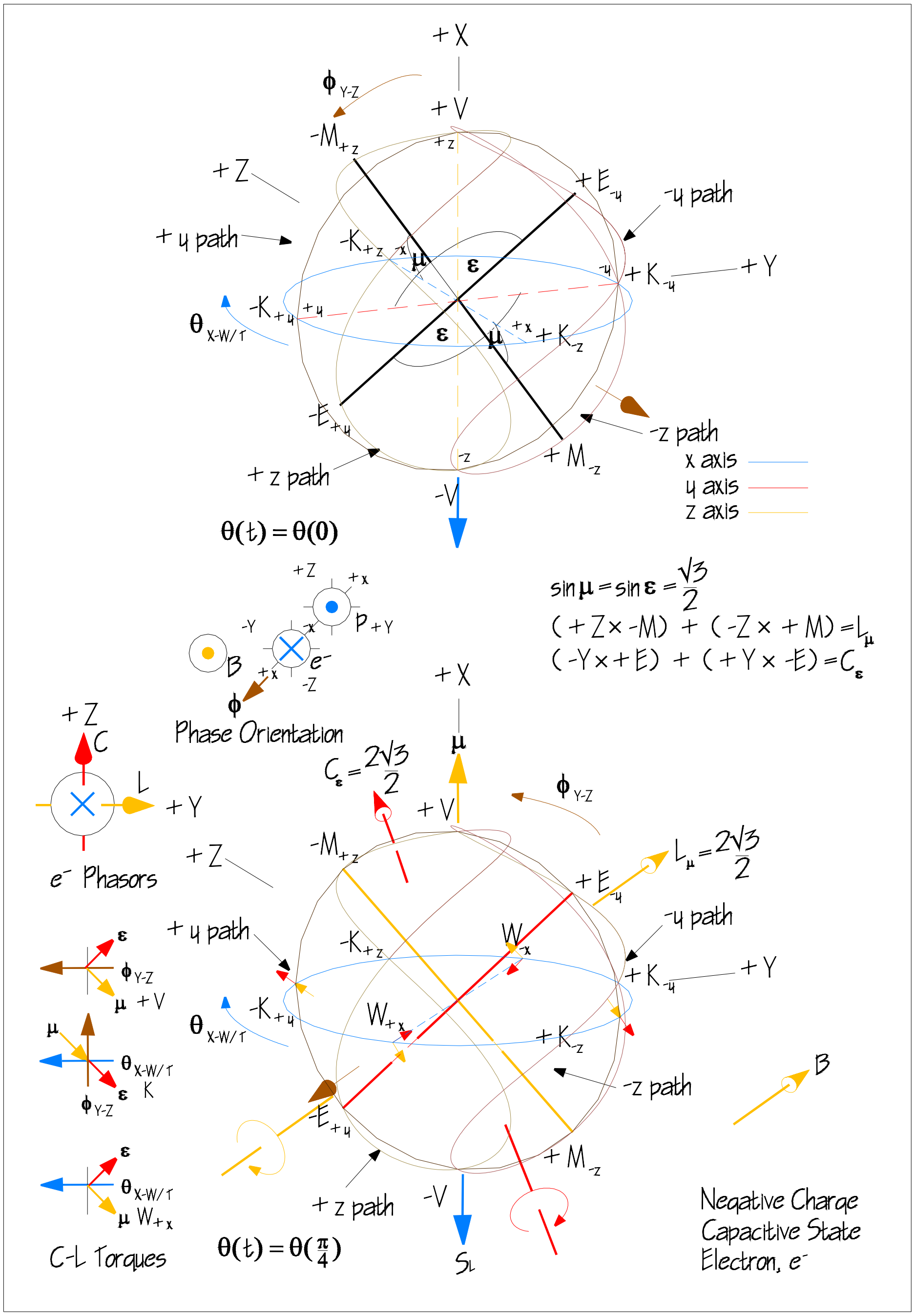
Spin Diagram 3 – Proton

The resulting condition, as shown in Spin Diagram 3, is that of the positive charge oscillation or the proton. The vantage point of the reader is shifted 90 degrees as indicated by the ϕ rotation vector which now aligns with the $-Y$ axis in the top figure. It is noted also that the power moments and their strain angles μ and ε have shifted $+i$ in Y-Z. In the bottom figure, the effect of this advance is to flip L_μ into the upper hemisphere where it continues to trail the capacitive vector by $-i$ and now trails W_{+x}. The result is a change in the shearing vectors at W_{+x} and W_{-x} such that the charge vector is now aligned with ϕ, with all the torques in general alignment with the rotation of θ, as indicated by S_L.

The phasor chart reflects this condition, with the capacitive vector continuing to lead the inductive, but with both in general alignment with the spin vector. The magnetic field vector B is now inverted from the position in Spin Diagram 2, to indicate that if activated as shown, L_μ would align with it and S_L and μ would precess around B, once again in the direction indicated by L_μ. This indicates that if our perspective of observation is stationed by the direction of B in the transition between Diagram 2 and 3, the result will be a reversal of the direction of θ and a flip of the spin and capacitive vectors. This can be seen with a little effort by viewing Diagram 3 upside down and "looking up" at θ.

As might be imagined, this indicates an instance of beta decay. The generation of a charge vector at W_{+x} involves the transmission of wave energy past the boundary node in the form of a <u>reflected</u> negatively charged oscillation which constitutes the electron as shown in Spin Diagram 4. The predominant amount of the energy is transmitted from the medium as potential to the oscillation as kinetic with the resulting altered resonant frequency of the proton. The quantification of this process will be shown in a minute.

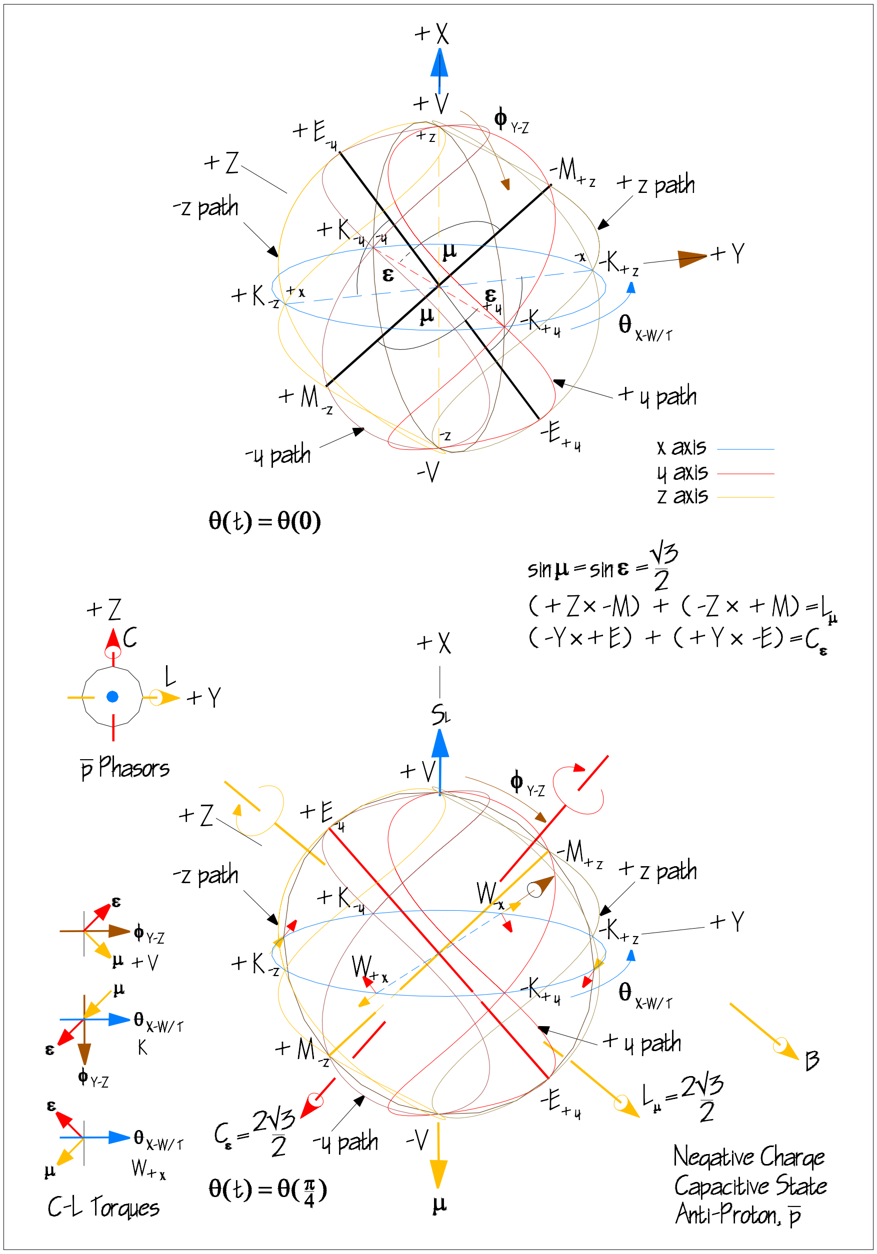
In Diagram 4, it should be noted that the reference grid shown is one half rotation from that shown in Diagram 3 to facilitate the viewing correspondences between the two. This is shown in the Phase Orientation chart at the left center of the diagram. Both diagrams share a common orientation for B. It is immediately apparent that the spin vector must flip from that of the proton as shown to provide continuity of the reflected wave. It is noted that the resulting oscillation involves an *inversion* of the y axis, while the x and z axes retain the same orientation with respect to their rest positions as does the neutral oscillation. Although Diagram 4 and related Diagram 6 are shown as spherical, the amplitude and inverse wave number for these oscillations are in fact not equal. While both amplitude and wave number and the covariant frequency are proportionally and equally smaller for the reflected oscillations, as differentials they have different physical interpretations. The amplitude is much smaller than that of the nuclear fundamental indicating a smaller size of the electron, while the equally smaller wave number that we will see is associated with the reflection would suggest a much elongated waveform with a proportionally reduced frequency. The inversion then indicates an inverted node, as the kinetic distal end of the reflected oscillation, which is encountered observationally as the electron orbital or cloud. The likelihood that this configuration involves an extension into a fourth-dimension should not be discounted, in which case the spherical form could be maintained.



Spin Diagram 4 - Electron

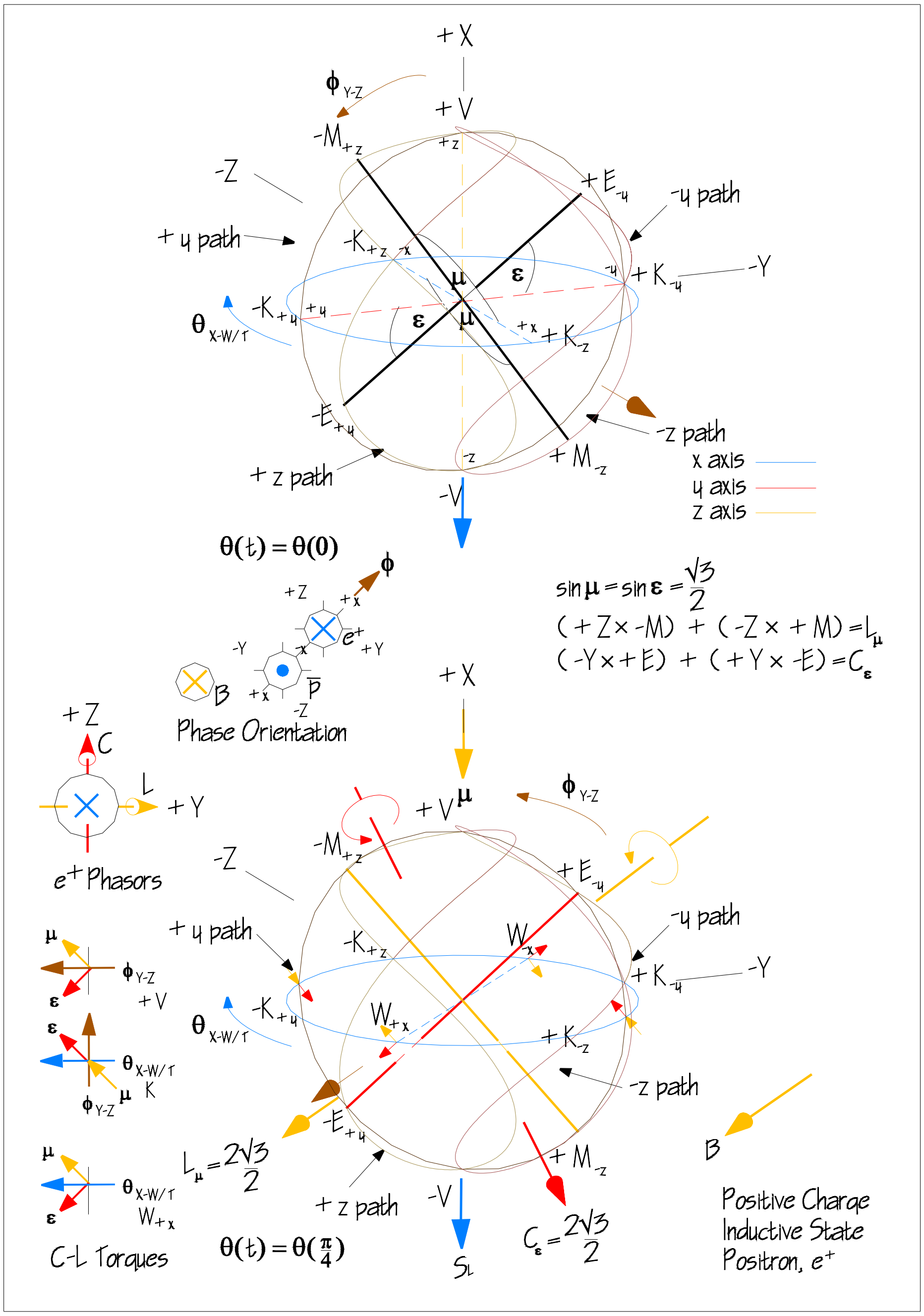
The shearing vectors all point *against* the direction of spin indicating a negative or receptive charge, a capacitive or charging state of the oscillation. As before in the presence of a magnetic field, the alignment of L_μ with B results in a precession of S_L and μ.

The subject of reflected and transmitted oscillations with charges reversed, i.e. anti matter, is straightforward. Cosmic expansion is clearly the conversion of potential to kinetic energy, the energy of position to that of motion. This could not be more evident than in the notion of a "big bang". The natural state for an oscillation under this condition is that of conversion of potential to kinetic energy, of electrical field potential to magnetically induced current. It is this condition that accounts for the experienced predominance of matter over anti-matter.



Spin Diagram 5 – Anti-Proton

Spin Diagram 5 shows the condition of a fundamental oscillation of the anti proton in the capacitive state resulting from the generation of a negative charge. Returning to Diagram 2 for a moment, if we cross the capacitive shear at W_{+x} and W_{-x} into the inductive, the resulting charge vector aligns parallel with the ϕ rotation vector, corresponding with a phase lag and eventual -*i* rotation of θ in the *Y-Z* plane, a long term physical *decrease* in the rotational strain in the direction of spin. It is a restoration of kinetic energy to potential energy over and above that normal to the oscillatory cycle.



Spin Diagram 6 – Positron

Note that the reader orientation of Diagram 5 is rotated from that of Diagram 2, ½ π in the opposite direction from that of the proton and is thus out of phase from it by a value of π. The resulting lag results in a flip of the capacitive torque to general alignment with the effective magnetic moment, μ, and an applied magnetic field, B. The shearing arrows now all point counter to the direction of spin, θ, indicating a charging or capacitive mode.

Spin Diagram 6 shows the state of the positron, a wave reflected at the boundary node at W_{-x} in Diagram 2. It is analogous to the electron except for the reversal of the capacitive and inductive vectors, and hence the charge. As a result, it is in an inductive state and the shearing vectors all point in general alignment with the rotation of θ.

With respect to the nomenclature of "reflected" and "transmitted" waves, they have been chosen for historical reasons having to do with the transmission of power of a traveling wave at a discontinuity. This treatment is used subsequently within the context of a 4 dimensional continuum. We can think of the neutron, proton and anti-proton as 3-dimensional transmissions of a 4-D wave, where the electron and positron are reflected as the result of an impedance change.

Generation of Charge

The following tables outline the dynamics of the quanta shown in Spin Diagrams 2 – 5 with respect to the generation of charge. The tangential and centripetal acceleration attributed to each of the torques C_ε and L_μ at the six nodes is given by the dot product, in the case of tangential, or the cross product, in the case of centripetal, of shear vectors ε and μ and the rotation vectors, θ and ϕ.

In these tables, the small circles, filled and unfilled, preceding the values in columns 4-7 refer to the centripetal or counter-centripetal sense respectively, arising from the cross product operation. The specific choice of the options of θ and ϕ in columns 1-2 and 5-6 is determined by context with respect to the "C-L Torques" charts as shown for each node[2] in the diagram designation column. Columns 1-2, 4-6 give the values of the products without modification by the value of the torque involved, while column 7 is so modified.

A cursory glance at the tables and the diagrams will provide ample indication of a quantum structure that has corollaries with the quark and chromodynamic modeling of the standard theory. In particular, a physical basis for fractional electrical charge suggests itself and that for asymptotic freedom is more assertive. The nodes, while hardly rigid, as aspects of a discrete wave would be anticipated to register and transfer to any observational media the interactions of inelastic collision and scattering. Obviously, however, they are inseparable as aspects of an overall oscillation, and hence convey the quality of "confinement".

In the spin diagrams, the tangential rotational path of θ, as distinct from its axial vector representation, S_L, is the locus of sustained kinetic energy, tangential momentum and velocity and the electrical equivalence, current. The E moments transfer this velocity/current via the elastic strain of the medium to the static point of maximum potential or electrical amplitude at $+V$ and $-V$, thus charging the electrical field of the system. (The whole oscillation is, of course, free to rotate in space, but for any short term series of oscillations, these points can be considered static.) The potential or electrical charge energy of $+V$ and $-V$ present in the strain is converted by recoil or restoration of that strain to kinetic or velocity/current via the M moments. The vectors ε and μ both point in the direction of increasing strain, but their direction over time is not the same.

We can envision three general energy states of the oscillation, resonant, R, capacitive, C, or inductive, L. In the resonant state, ε points universally in the direction of the rotation tangent vectors, while μ points counter to those vectors, indicating the characteristic phase lead of ε and lag of μ for current with respect to potential. In columns 1 and 2, the dot product of these vectors and the rotation vectors cancel, indicating no net change or acceleration of the latter and a general condition of resonance, as indicated in column 3.

[2] As determined by context, the term *node* is used as a general term to include the category of *antinode*.

	1	2	3	4	5	6	7	8
Diagram, Node/Antinode, Rotation	$\mu\cdot$ Into θ,ϕ	$\cdot\varepsilon$ From θ,ϕ	K-V State	$S = \mu \times \varepsilon$ $\mu,\varepsilon = \frac{\sqrt{2}}{\sqrt{3}}$	$\mu\times$ Into θ,ϕ	$\times\varepsilon$ From θ,ϕ	$q = \frac{S}{\lvert S \rvert}\left(\mathrm{T}\left(\mu\times+\times\varepsilon\right)\right)$ $\mathrm{T}_{\mu,\varepsilon} = \frac{\sqrt{3}}{4}$	Total Charge $q_{W-} - q_{W+}$ $q_{V\pm} - q_{V\mp}$
Diagram 2 - Neutron								
W_{+x}-θ	$-\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	R	$\circ\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{4}+\bullet\frac{1}{4}=0$	
W_{-x}-θ	$-\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	R	$\bullet\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{4}+\circ\frac{1}{4}=0$	0
K_o -θ	$-\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	R	0	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
K_π -θ	$-\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	R	0	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
K_o -ϕ	$-\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	R	0	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
K_π -ϕ	$-\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	R	0	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
$+V$ -ϕ	$-\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	R	$\circ\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{4}+\bullet\frac{1}{4}=0$	
-V -ϕ	$-\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	R	$\bullet\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{4}+\circ\frac{1}{4}=0$	[0]
Diagram 3 - Proton								
W_{+x}-θ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	$\bullet\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet 1\left(\bullet\frac{1}{4}+\bullet\frac{1}{4}\right)=+\frac{1}{2}$	
W_{-x}-θ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	$\circ\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\circ 1\left(\circ\frac{1}{4}+\circ\frac{1}{4}\right)=+\frac{1}{2}$	+1
K_o -θ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	0	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
K_π -θ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	0	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
K_o -ϕ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	0	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
K_π -ϕ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	0	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
$+V$ -ϕ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	$\circ\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ 1\left(\bullet\frac{1}{4}+\bullet\frac{1}{4}\right)=-\frac{1}{2}$	
-V -ϕ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	$\bullet\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet 1\left(\circ\frac{1}{4}+\circ\frac{1}{4}\right)=-\frac{1}{2}$	$[-1]=-i1$
Diagram 4 - Electron								
W_{+x}-θ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	$\circ\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ 1\left(\bullet\frac{1}{4}+\bullet\frac{1}{4}\right)=-\frac{1}{2}$	-1
W_{-x}-θ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	$\bullet\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet 1\left(\circ\frac{1}{4}+\circ\frac{1}{4}\right)=-\frac{1}{2}$	
K_o -θ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	0	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
K_π -θ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	0	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
K_o -ϕ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	0	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
K_π -ϕ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	0	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
$+V$ -ϕ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	$\circ\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ 1\left(\bullet\frac{1}{4}+\bullet\frac{1}{4}\right)=-\frac{1}{2}$	$[-1]=-i1$
-V -ϕ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	$\bullet\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet 1\left(\circ\frac{1}{4}+\circ\frac{1}{4}\right)=-\frac{1}{2}$	

Spin Table 1 – Inductive State, Ordinary Matter

	Diagram, Node/Antinode, Rotation	1	2	3	4	5	6	7	8
		$\varepsilon\cdot$ Into θ,ϕ	$\cdot\mu$ From θ,ϕ	K-V State	$S = \varepsilon \times \mu$ $\mu,\varepsilon = \frac{\sqrt{2}}{\sqrt{3}}$	$\varepsilon\times$ Into θ,ϕ	$\times\mu$ From θ,ϕ	$q = \frac{S}{\lvert S\rvert}\left(\mathrm{T}\left(\mu\times+\times\varepsilon\right)\right)$ $\mathrm{T}_{\mu,\varepsilon} = \frac{\sqrt{3}}{4}$	Total Charge $q_{W-} - q_{W+}$ $q_{V\pm} - q_{V\mp}$
Diagram 2 – Anti Neutron	W_{+x} - θ	$+\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	R	$\bullet\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{4}+\bullet\frac{1}{4}=0$	
	W_{-x} - θ	$+\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	R	$\circ\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{4}+\circ\frac{1}{4}=0$	0
	K_o - θ	$+\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	R	0	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
	K_π - θ	$+\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	R	0	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
	K_o - ϕ	$+\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	R	0	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
	K_π - ϕ	$+\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	R	0	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
	$+V$ - ϕ	$+\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	R	$\bullet\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{4}+\bullet\frac{1}{4}=0$	
	$-V$ - ϕ	$+\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	R	$\circ\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{4}+\circ\frac{1}{4}=0$	[0]
Diagram 5 – Anti Proton	W_{+x} - θ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	$\circ\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\circ 1\left(\bullet\frac{1}{4}+\bullet\frac{1}{4}\right)=-\frac{1}{2}$	
	W_{-x} - θ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	$\bullet\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\bullet 1\left(\circ\frac{1}{4}+\circ\frac{1}{4}\right)=-\frac{1}{2}$	-1
	K_o - θ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	0	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
	K_π - θ	$-\frac{1}{\sqrt{3}}$	$-\frac{1}{\sqrt{3}}$	C	0	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
	K_o - ϕ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	0	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
	K_π - ϕ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	0	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
	$+V$ - ϕ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	$\bullet\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet 1\left(\bullet\frac{1}{4}+\bullet\frac{1}{4}\right)=+\frac{1}{2}$	
	$-V$ - ϕ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	$\circ\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\circ 1\left(\circ\frac{1}{4}+\circ\frac{1}{4}\right)=+\frac{1}{2}$	$[+1]=+i1$
Diagram 6 - Positron	W_{+x} - θ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	$\bullet\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet 1\left(\bullet\frac{1}{4}+\bullet\frac{1}{4}\right)=+\frac{1}{2}$	+1
	W_{-x} - θ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	$\circ\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\circ 1\left(\circ\frac{1}{4}+\circ\frac{1}{4}\right)=+\frac{1}{2}$	
	K_o - θ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	0	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
	K_π - θ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	0	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
	K_o - ϕ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	0	$\bullet\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	0	
	K_π - ϕ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	0	$\circ\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	0	
	$+V$ - ϕ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	$\bullet\frac{2}{3}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet\frac{1}{\sqrt{3}}$	$\bullet 1\left(\bullet\frac{1}{4}+\bullet\frac{1}{4}\right)=+\frac{1}{2}$	$[+1]=+i1$
	$-V$ - ϕ	$+\frac{1}{\sqrt{3}}$	$+\frac{1}{\sqrt{3}}$	L	$\circ\frac{2}{3}$	$\circ\frac{1}{\sqrt{3}}$	$\circ\frac{1}{\sqrt{3}}$	$\circ 1\left(\circ\frac{1}{4}+\circ\frac{1}{4}\right)=+\frac{1}{2}$	

Spin Table 2 – Capacitive State, Anti Matter

Note that there is no physical difference between the neutron and the anti neutron, in which the capacitive and inductive strains are in equilibrium, though this oscillation appears in each table to reflect the underlying symmetry of the system. In Spin Table 1, it is embedded in an overall inductive state, as indicated by the vector operations at the top of each column, while in Spin Table 2 it is in an overall capacitive scheme.

In the case of the cross product operations, the results are shown with the novel sense to facilitate the algebra with respect to the spin surface. The crossing is done in the order outlined above for the inductive state of ordinary matter, that is, from μ to ε, θ or ϕ and from θ or ϕ to ε, and for the capacitive state of anti matter, from ε to μ, θ or ϕ and from θ or ϕ to μ. In this algebra, we have:

$$\circ 1\left(\circ 1+\circ 1\right)=+2 \tag{2.130}$$

$$\bullet 1\left(\bullet 1+\bullet 1\right)=+2 \tag{2.131}$$

$$\circ 1\left(\bullet 1+\bullet 1\right)=-2 \tag{2.132}$$

$$\bullet 1\left(\circ 1+\circ 1\right)=-2 \tag{2.133}$$

In the case of the neutron, the cross products at each of the W nodes cancel, resulting in no net change in the oscillatory state, potential/kinetic, electrical/magnetic. In the case of the proton, there is a net inductive change or positive charge along the axis of ϕ, and a corresponding capacitive change or negative charge along the axis of ϕ of the electron. In the case of the anti-proton, there is a net capacitive change or negative charge along ϕ, and a corresponding inductive change or positive charge along ϕ of the positron. Thus the sense of the charge is positive if $\mu\times\varepsilon$ is parallel to $\left(\mu\times+\times\varepsilon\right)$ and negative if they are anti-parallel, the positive charge representing an advance of the tangential momentum, or inductance, and the negative charge representing a retention of that momentum, capacitance. When $\mu\times$ is of opposite sense to $\times\varepsilon$, they cancel, there is no net charge and a state of resonance is maintained.

For the proton-electron pair and the inductive state, there is also a corresponding charge for each along the axial vector, S_L. Since these rotational and torque vectors are anti-parallel, indicating a negative charge for the pair, according to the C-L torque diagrams, the charge for both is *–i1*. They are complex, indicating their orthogonality. For the capacitive state, charge summation is from the same potential nodes, but the spins, relative to ε and μ, are reversed, leading to a reversal of charges or *+i1* for the anti proton and positron.

The square brackets for these charges indicate that they pertain to the $+V$ and $-V$ potential nodes, which with respect to the ϕ oscillation are antinodes as are K_o and K_π. These former are the general loci of the wave boundary of θ, which has been rotated into the Y-Z plane and forms the wave boundary and nodes of ϕ at W_+ and W_-. Any transmission of momentum and energy through the boundary at W_+ and W_- would presumably involve the perpetual strain between the X and x axes and thereby the $+V$ and $-V$ nodes. The charges indicated at the latter, therefore, are not additional but are rather the complex expression of those at W_-. It should be noted that while we have assigned the charge to W_- for descriptive facility, the same charge can be applied to the opposite node. We would imagine, however, that with beta decay the reflected oscillation which becomes the electron occurs at one or the other end of the fundamental *x* axis, i.e. in the vicinity of -V or +V, along the X axis. The direction of the axial vector ϕ is not necessarily a directional vector for electron reflection, but rather the indicator of ϕ spin for both particles at the time of transmission. This wave transmission/reflection results from a conversion of the potential energy density of the continuum to the kinetic energy of proton-electron oscillation through a decrease in the continuum impedance. It should not be ruled out that the electron wave in fact forms a spherical shell about the oscillation source, emanating from the two ends at -V and +V, with its amplitude as developed at (3.34) , and the shell as generally described in Spin Diagram 4.

K_o and K_π are antinodes for both ϕ and θ and not candidates for the longitudinal energy and momentum transmission of beta decay. This is indicated by the vanishing cross products, $\mu \times \varepsilon$ and $\varepsilon \times \mu$. The products of $\times\varepsilon$ and $\mu\times$ (and their anti matter counterparts), while not vanishing, cancel at each node, ϕ to θ for the resonant state, ε to μ for the proton and anti proton, and with respect to both for the electron and positron. This last condition is a reflection of the apparent lack of internal structure of these last two particles. The strain vectors ε and μ at all of the nodes are capacitive for the electron and inductive for the positron, resulting in an isotropic charge condition registering from particle interaction. In the case of the neutron they are balance between capacitance and inductance at each node. In that of the proton and anti proton one or the other predominates, and some indication of internal structure can be registered from collision and scattering.

Since a charge can be assigned to either node of ϕ or θ, we might surmise that the fundamental angular frequency, ω_0, associated with the tangential momentum be divided by π. Thus we can state a preliminary or raw quantification of the charge generated by the oscillation resulting in the process of beta decay, in light of (2.127) as a transfer of linear momentum or

$$e \approx \frac{1}{\pi} 4 \left| \left(\left(\frac{\sqrt{2}}{\sqrt{3}} \right) \frac{\sqrt{3}}{4} \hbar c \right) \times \frac{1}{\sqrt{2}} \kappa_0 \right| = \frac{\pm i \hbar \omega_0}{\pi} = \pm i 2 \hbar \nu_0 \tag{2.134}$$

where ν_0 is the cyclic frequency of the oscillation. Conceptually, this reflects the fact that for each cycle of the fundamental oscillation, there are two phases of capacitance and two of inductance, two antinodes of maximum charge and two antinodes of maximum current. The ambiguous sense indicates the oscillation through both semi-cycles. Evaluation and comparison with CODATA observation is at (6.4) and (6.28). If we make this an equation, based on this data, we have

$$e = 1.002405818\ldots \left(\frac{\pm i \hbar \omega_0}{\pi} \right) \approx \left(1 + Z_0\right) \frac{\pm i \hbar \omega_0}{\pi} = \left(1 + \hbar \kappa_0 \omega_0\right) \frac{\pm i \hbar \omega_0}{\pi} \tag{2.135}$$

$$\approx 1.002390877\ldots \left(\frac{\pm \hbar \omega_0}{\pi} \right) = \pm \frac{1}{\pi} \left(\hbar \omega_0 + \hbar^2 \omega_0^2 \kappa_0 \right) = \pm \frac{1}{\pi} \left(G_0 + G_0^2 \kappa_0 \right) \tag{2.136}$$

where Z_0 is the mechanical impedance of the continuum and G_0 is the transverse wave momentum. Addition of this factor is off the observed value by a factor of 0.000014905…which we might compare with (1.12) at 0.000014648….

Pursuing this a bit further, we have

$$e^2 \approx \frac{\left(\pm i\sqcap\omega_0\right)^2}{\pi^2} = -\frac{\sqcap\left(\sqcap\omega_0^2\right)}{\pi^2} = \sqcap\frac{\lambda_0 c^2}{\pi^2} = \hbar\frac{Z_0}{\pi^2} = \frac{Z_0^2}{\pi^2\left(i\kappa\right)_0^2}. \tag{2.137}$$

In the next to last term, the inertial density times c is the mechanical impedance of the continuum. In light of our prior discussion concerning the monad, or differential horn torus, it bears noting that if we transpose the π^2 term to the left hand side, we have the form of the equation for the surface area of the horn torus.

Further development, using the familiar identity for the fine structure constant

$$\alpha^{-1} = \hbar c\frac{4\pi\varepsilon_0}{e^2} \tag{2.138}$$

and the permeability, μ_0, and permittivity, ε_0, relationship

$$\varepsilon_0 = \frac{1}{c^2\mu_0} \tag{2.139}$$

shows

$$e^2 = -\sqcap c^2\left(\alpha 4\pi\varepsilon_0\right) = -\sqcap\frac{\alpha 4\pi}{\mu_0} = -\sqcap\frac{\alpha}{10^{-7}} \approx -\sqcap\frac{\lambda_0 c^2}{\pi^2}. \tag{2.140}$$

It is noted that the value of μ_0 is set by convention for computational facility in relating charge, q, (of which elementary charge, e, is an effective quantum) and current, $i = dq/dt$, resulting in the exactness of the denominator of the next to last term. Since the negative sense of the right terms above can be attributed to the current squared, it can be incorporated therein, canceling such sense in the charge squared term. With reference to Appendix A, this suggests the transparent presence of a current squared argument in (2.140), for which the fine structure constant is a coefficient, since for one ampere^2 of current, where the denominator on the right is in Newton,

$$\frac{4\pi}{\mu_0} = \frac{i^2}{10^{-7}}. \tag{2.141}$$

Thus (2.140) becomes

$$e^2 = \sqcap\frac{\alpha}{10^{-7}}i^2 \tag{2.142}$$

Given the dimensions of $\sqcap$, and since 10^{-7} has the presumed units of force, the fine structure constant then, is dimensionless, yet may be the ratio of two forces. If e has the units of momentum, then this would necessarily be the case and current would have the units of force, as $F = dP_M/dt$. Since the value of 10^{-7} is conventional, we would like to convert it to some natural expression of force, presumably related to cosmic expansion, as

$$\left(\frac{e}{i}\right)^2 = \hbar\frac{\alpha}{10^{-7}} = \hbar\alpha' = \hbar\frac{k^2}{\hbar\omega_e^2}, \tag{2.143}$$

where we have transposed the current to give the square of the ratio of elementary charge to current and which will structure α' as

$$\alpha' = \frac{k^2}{\hbar\omega_e^2} = \frac{\alpha}{10^{-7}}. \tag{2.144}$$

In combination with (2.134), (in which it is assumed that the preliminary factor of π^{-2} is included in k^2) this gives

$$\left(\frac{e}{ki}\right)^2 = \frac{\hbar}{\hbar\omega_e^2} = \frac{1}{\omega_e^2} = \frac{1}{\partial\omega_0^2} \tag{2.145}$$

in which the coefficient k is a normalizing factor, so that inverting we have the frequency differential arising from cosmic expansion and responsible for the generation of charge

$$\partial\omega_0 = k\frac{i}{e} = \omega_e. \tag{2.146}$$

This suggests and another look reveals that (2.140) is in fact a differential equation in which

$$e^2 = \left(\frac{\partial i}{\partial\omega_0}\right)^2 = \hbar\alpha' = \hbar\left(7.297352568...x10^4\right)\ C^2/\theta^2, \text{ thus} \tag{2.147}$$

$$e = \frac{\partial i}{\partial\omega_0} = 270.1361239\sqrt{\hbar}\ C/\theta \tag{2.148}$$

3 - Cosmic Expansion as the Driving EMF of a Quantum State

The Reactance States and Electron-Positron Generation

In light of the above correspondence between the potential-kinetic and electrical-magnetic energy cycles of the quantum spin, the cosmic expansion can be seen to provide a driving electromotive force or emf, $\mathcal{E}$, which is necessarily tuned to the natural or resonant angular frequency of any local section of the 3-D cosmos. "Necessarily" indicates that the 3-D tension in the 3-space surface of an expanding 4-core is a function of the inertial density of that 4-core, expressed as either energy or mass density as

$$T = \text{ת}''(\kappa, \omega) = {}_2\text{ת}_2 = {}_4\text{ת}c^2 , \tag{3.1}$$

where the volume inertial density, ρ_0, is

$$\rho_0 = -\lambda_0 \kappa_0^2 = \text{ת}\kappa_0^4 = {}_4\text{ת} \quad . \tag{3.2}$$

While the 4-core appears to have elastic properties, in keeping with (2.101) the decrease in inertial density arising from expansion leads to a decrease in 3-D tension. At the boundaries of the oscillation, W_{+x} and W_{-x}, this results in a differential <u>increase</u> in the transverse wave speed and a differential increase in the amplitude A for ϕ, with a concomitant contraction of the W_{+x}-W_{-x} axis and differential increase in the wave number, κ_0, in keeping with the gravitational quantum of (2.107). If c is invariant, this indicates a corresponding increase in ω_0. We have a paradox of sorts due to (2.101), however, in that a decrease in the inertial density requires a decrease in the wave force and of ω_0 over time, so that ω_0 at some future time $t(p)$, becomes $\omega_0 \to \omega_p$, where $\omega_0 > \omega_p$ for an inductive cosmic state, i.e. that of general expansion.

While the continuum itself, in the field, and the oscillation in a state of resonance will retain the invariance of c, so that as functions of time $c(t)$,

$$c = c(0) = \frac{\omega_0}{\kappa_0} = c(p) = \frac{\omega_p}{\kappa_p} \tag{3.3}$$

we might surmise that within the boundary of the driven oscillation itself

$$c = \frac{\omega_0}{\kappa_0 \pm \delta\kappa} = \frac{\omega_0 \pm \delta\omega}{\kappa_0} , \tag{3.4}$$

where the delta indicates a variation in the frequency or wave number.

With respect to a driving emf in an RLC circuit, the amplitude of the emf, $\mathcal{E}$, is related to the current amplitude, I, by the impedance, Z, as

$$I = \frac{\mathcal{E}}{Z} \tag{3.5}$$

where the impedance is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} \tag{3.6}$$

and R is the resistance, X_L is the inductive reactance and X_C is the capacitive reactance.

In a condition of resonance, represented by Spin Diagram 2,

$$X_L - X_C = 0 \ \left(\text{Resonant State}\right) \tag{3.7}$$

and the impedance is equal to R and to the mechanical impedance of (2.91),

$$Z = Z_0 = \hbar\omega_0\kappa_0 = \frac{\hbar\omega_0^2}{c} = \lambda_0 c = R \tag{3.8}$$

Since we might reason that $\mathcal{E} = \hbar\omega_0^2$, (3.5) in a resonant condition can be expressed as

$$I = c = \frac{\hbar\omega_0^2}{\hbar\omega_0\kappa_0} = \frac{\omega_0}{\kappa_0} \tag{3.9}$$

where the current amplitude is equal to the speed of wave propagation.

The inductive reactance, X_L, is equal to the product of the driving frequency, $\omega_d = \omega_0$, and the inductance, L, while the capacitive reactance, X_C, is equal to the inverse product of the driving frequency, ω_0, and the capacitance, C, or

$$X_L \equiv \omega_0 L \text{ and} \tag{3.10}$$

$$X_C \equiv \frac{1}{\omega_0 C}\,. \tag{3.11}$$

The inductance then is the ratio of the wave momentum per wave velocity to the changed impedance of the expanding medium, arising from a change in resonant frequency squared, or changed force per wave velocity,

$$L = \frac{\hbar\omega_0 c^{-1}}{\hbar\omega_p^2 c^{-1}} = \frac{P_{M0} c^{-1}}{\tau_p c^{-1}}\,. \tag{3.12}$$

The capacitance is the ratio of the fundamental mass as determined by the inertial density of the resonant and expanding medium to the impedance of the driving emf, or

$$C = \frac{\hbar\kappa_p}{\hbar\omega_0\kappa_0} = \frac{m_p}{Z_0} \tag{3.13}$$

Therefore the inductive reactance, the ratio of the driving impedance to the decreased impedance of expansion, of the driving frequency squared to the decreased frequency squared, and of initial acceleration to subsequent acceleration, is

$$X_L = \frac{\hbar\omega_0^2 c^{-1}}{\hbar\omega_p^2 c^{-1}} = \frac{Z_0}{Z''(\omega)} = \frac{\omega_0^2}{\omega_p^2}. \tag{3.14}$$

The capacitive reactance, the ratio of the driving impedance to the decreased impedance due to a change in wave number or the ratio of initial to subsequent wave numbers, is

$$X_C = \frac{\hbar\omega_0\kappa_0}{\hbar\omega_0\kappa_p} = \frac{Z_0}{Z'(\kappa)} = \frac{\kappa_0}{\kappa_p} \tag{3.15}$$

For an inductive state indicated by the above,

$$X_L \equiv \omega_0 L > \frac{1}{\omega_0 C} \equiv X_C, \tag{3.16}$$

while for a capacitive state,

$$X_L \equiv \omega_0 L < \frac{1}{\omega_0 C} \equiv X_C \tag{3.17}$$

For an expanding section of the cosmos, we would expect an inductive state to predominate; in a contracting section, we would expect a capacitive state.

We might anticipate that for such an inductive state the following would be found for a half spin oscillation,

$$\begin{aligned} \frac{\sqrt{3}}{2} I &= \frac{\mathcal{E}}{\sqrt{Z_0^2 + \left(X_L - X_C\right)^2}} \\ \frac{\sqrt{3}}{2} c &= \frac{\hbar\omega_0^2}{\sqrt{\left(\hbar\omega_0\kappa_0\right)^2 + \left(\frac{\omega_0^2}{\omega_p^2} - \frac{\kappa_0}{\kappa_p}\right)^2}} \end{aligned} \tag{3.18}$$

Solving with the CODATA values of *c*, $\kappa_0 = \frac{m_n}{\hbar}$, and $\omega_0 = c\kappa_0$ per the evaluation section gives

$$X_L - X_C = 1.380373411\ldots x10^{-3} \tag{3.19}$$

Some further algebra finds the following dimensionless ratios for the change in ω and κ of

$$R_m = \frac{\omega_0}{\omega_p} = \frac{\kappa_0}{\kappa_p} = \frac{\hbar\kappa_0}{\hbar\kappa_p} = \frac{m_n}{m_p} = 1.0013784732\ldots \tag{3.20}$$

where the third ratio is that of the product of the inertial constant and the wave numbers of the driving oscillation and the driven resonant oscillation and the fourth is that of the mass of the neutron to that of the proton.

The deviation of this derived theoretical ratio from the CODATA observed ratio is

$$R_{m\ theoretical} - R_{m\ observed} = 1.0013784732\ldots - 1.00137841870(58) = 5.45x10^{-8}. \tag{3.21}$$

While this is slightly outside the relative standard uncertainty indicated for the observed figure, when it is coupled with the broad uncertainty assigned to the gravitational constant, which no doubt enters systemically into the observed computation and its deviation from the theoretical value herein arrived at of 0.000015019, as per (1.12) the correlation is significant.

Of related interest is the following, where

$$\Delta_{\omega_0} = R_{m\ t} - 1 = \frac{\omega_0 - \omega_p}{\omega_p} = 0.0013784732\ldots \tag{3.22}$$

so that the quotient of the mechanical impedance and the relative change in fundamental frequency, $\Delta_{\omega 0}$, times $R_{m\ t}$ is

$$\frac{Z_0}{R_{m\ t}\left(R_{m\ t} - 1\right)} = \frac{\omega_p}{\Delta_{\omega_0}} \sqcap \kappa_0 = \frac{\omega_p}{\Delta_{\omega_0}} m_0 = 1.732050837\ldots \cong \sqrt{3} \tag{3.23}$$

within a deviation of $2.9x10^{-8}$. An identical result applies to the corresponding operation of the wave number, κ. The appearance of $\sqrt{3}$ once again suggests a 4-D relationship.

The interpretation is that in the context of certain sufficient conditions of nuclear congregation and the confines of an inertial sink or font, the fundamental oscillation of frequency ω_0 and wave number κ_0 has an impedance matching the driving emf of cosmic expansion. These fundamentals decrease slowly, but exponentially over time with cosmic expansion in keeping with the driving frequency, ω_d, which, along with the mechanical impedance, Z_0, is a function of decreasing inertial density, λ_0, of an expanding cosmic 4-core.

Absent such congregation or confinement, the fundamental oscillation responds as a driven wave to the resonant frequency of the local continuum, ω_p, which, along with the mechanical impedance, Z_p, is a function of the exponentially decreasing inertial density, λ_p, of the expanding cosmic 3-surface. We can think of the differential between ω_d and ω_p as either a time or space gradient, as it is a spacetime gradient. The coefficient for the current amplitude of (3.18) arises from the rotational dynamics of the oscillation as outlined above, itself an expression of the 4-d orthogonality of those dynamics.

While this accounts for the energy of the proton, we still have that of the electron to include in the equation. We would anticipate that along with the transmitted energy of the proton, once again as a transitional state or conversion of the potential energy of inertial density to the dynamics of oscillation, there is a reflected component that accounts for the electron, as well as perhaps some other component that is accounted for by the neutrino in the standard model.

With reference to the Appendix – Wave Transmission at a Discontinuity, which is a one dimensional ideal string model, since our equations make use of the linear inertial density of the medium we can use the final impedance terms in the equations to solve for the complex amplitude reflection, $\breve{R}_a$, and transmission, $\breve{T}_a$, coefficients or ratios, and for the power reflection, R_p, and transmission, T_p, coefficients, which depend only on the properties of the medium. The complex amplitude reflection and transmission coefficients are applied to the complex modulus of the fundamental amplitude. The impedance for the driving cosmic core, Z_0, and expanding surface, Z_p, from above are

$$Z_0 = Z_1 = \text{ח}\omega_0\kappa_0 = 0.002390876881...kg \cdot \theta / s \tag{3.24}$$

$$Z_p = Z_2 = \text{ח}\omega_p\kappa_p = 0.002384298968...kg \cdot \theta / s\,. \tag{3.25}$$

and the above coefficients are

$$\breve{R}_a = \frac{Z_1 - Z_2}{Z_1 + Z_2} \tag{3.26}$$

$$\breve{T}_a = \frac{2Z_1}{Z_1 + Z_2} \tag{3.27}$$

$$R_p = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2 \tag{3.28}$$

$$T_p = \frac{4Z_1Z_2}{\left(Z_1 + Z_2\right)^2} \tag{3.29}$$

resulting in the following evaluations.

$$\breve{R}_a = 0.001377522755... \tag{3.30}$$

$$\breve{T}_a = 1.001377522755... \tag{3.31}$$

$$R_p = .000001897568941... \tag{3.32}$$

$$T_p = 0.9999981024... \tag{3.33}$$

Applying the dimensionless coefficient, $\breve{R}_a$ to the amplitude = inverse wave number of the neutron, gives us an amplitude of the electron, A_e, as the reflected oscillation of

$$A_e = \breve{R}_a \frac{1}{\kappa_0} = \breve{R}_a \frac{\text{ח}}{m_n} = 2.893065241...x10^{-19}\, m\,. \tag{3.34}$$

Since the proton has no established decay rate, and we have shown that its energy, therefore its frequency, is a direct function of a decreasing cosmic inertial density, we might anticipate that $\breve{R}_a$, as with the other coefficients, is a differential or derivative of that decrease and of expansion. With respect to (2.145) through (2.148) we still do not have a value for (2.146) or k itself, though we can find it empirically by the ratio of neutron to electron mass, which according to the CODATA values is

$$\frac{m_n}{m_e} = 1838.6836598(13)\,. \tag{3.35}$$

We would like to find some mechanism, both on a quantum or intrinsic and cosmic or extrinsic level that would make the determination, since it is the value of this differential that determines k and α'. We discussed this previously at (2.55) to (2.57). Recapitulating from that discussion, the variation of the unit area arising from a change in expansion stress over time is

$$\delta = \delta \mathrm{A} = \delta r^2 = 0.0005441353061\ldots. \tag{3.36}$$

The first delta is a coefficient which references the number magnitude without any applicable dimensions or units. The variance in the stress manifests itself in a fluctuation in the fundamental frequency and wave number of the oscillation over time within a limit that precludes transmission through the boundary. At the time such limit is exceeded and transmission occurs, therefore, we would anticipate the following ratio and range to hold, shown here for the wave number, but equally applicable to the frequency,

$$0.0005438393\ldots < \frac{\delta\kappa}{\kappa_0 \pm \delta\kappa} < 0.0005444315\ldots. \tag{3.37}$$

While in linear proportion to the variance, the change in wave number would tend to be negative in an expanding medium or toward a smaller number, that is to a lower ω and a proportionally lower κ, if c is invariant.

With reference to the energy of the system, we might imagine that while over time the kinetic and potential energy in resonance is balanced, at any instant the above variance may be in play so that the Lagrangian of the fundamental oscillation, where the kinetic and potential components cancel, is

$$\mathcal{L} = \left(K_\phi - V_\phi\right) \pm \delta E = \left(0\right) \pm \delta E \tag{3.38}$$

The Hamiltonian or total energy of the system, then is

$$H = 2K_\phi - \mathcal{L} = 2K_\phi \mp \delta E = K_\phi + V_\phi \mp \delta E \tag{3.39}$$

so that

$$\begin{aligned} H - \delta E = 2K_\phi - \delta E &= E_0 + E_p + \Delta E_{0-p} - \delta E_0 \\ &= E_0 + E_p + \left(\Delta_{0-p} - \delta\right) E_0 \\ &= \hbar\kappa_0 c^2 + \hbar\kappa_p c^2 + \left(\Delta_{0-p} - \delta\right)\hbar\kappa_0 c^2 \\ &= \hbar c\omega_0 + \hbar c\omega_p + \left(\Delta_{0-p} - \delta\right)\hbar c\omega_0 \end{aligned} \tag{3.40}$$

The variation on the left hand side represents the differential expansion energy, the first two terms on the right side are the neutron and proton mass-energy equivalence and the bracketed term is their difference. Since the amplitude and the inverse wave number appear to be equated in the case of the fundamental quantum oscillation, we might imagine an analogy with respect to the reflected wave, with the understanding that the smaller amplitude indicates a smaller wave number for the reflection. Thus substituting the reflection amplitude coefficient for the difference in wave number/angular frequency, and using the variance from (3.36), gives a difference of

$$\begin{aligned} \Delta_{0-p} - \delta \simeq \breve{R}_a - \delta &= 0.001377522755\ldots - 0.0005441353061\ldots \\ &= 0.0008333874489\ldots \end{aligned} \tag{3.41}$$

From another approach, with respect to (7.139) in the appendix on exponentiation, we might surmise that the acceleration normalizing factor represents a change in the Hamiltonian due to a dilative change in the time scale associated with beta decay,

$$ie_2 = \frac{\partial H}{\partial t_\beta} = i1.531584397... \tag{3.42}$$

so that canceling the δE terms from both sides, we have

$$H = \hbar\kappa_0 c^2 + \hbar\kappa_p c^2 + \left(1 + ie_2\right)\delta' E_0 \text{ and} \tag{3.43}$$

$$H = \hbar c\omega_0 + \hbar c\omega_p + \left(1 + ie_2\right)\delta' E_0 \tag{3.44}$$

where

$$\delta' = \frac{\breve{R}_a}{\left(1 + ie_2\right)} \tag{3.45}$$

With evaluation, we have

$$\delta' = 0.0005441346363... \tag{3.46}$$

$$ie_2\delta' = i0.0008333881188... \tag{3.47}$$

where apparently

$$ie_2\delta' = i\left(\Delta_{0-p} - \delta\right) \tag{3.48}$$

Note that δ is strictly a geometric derivation from (3.36) with no direct connection to the derivation of δ' from (3.30), and that their difference is

$$\Delta\delta = \left(\delta - \delta'\right) = 6.7x10^{-10}, \tag{3.49}$$

a significant correlation. A similar condition results where $ie_2\delta'$ is derived conceptually using (7.135) and (3.45) and independent of $\Delta_{0-p} - \delta$.

This suggests that the following are invariants of the system, where

$$\delta = \delta' \text{ and} \tag{3.50}$$

$$\left(1 + ie_2\right) = \frac{\Delta_{0-p}}{\delta} = 2.531584397.... \tag{3.51}$$

From (3.39) and (3.40), where κ_e is the wave number of the reflected wave, we have the following equations in terms of potential (3.52) and kinetic (3.53) energy,

$$\hbar\kappa_0 c^2 - \Delta_{0-p}E = \hbar\kappa_0 c^2 - \hbar c^2\left(1 + ie_2\right)\delta\kappa_0 = \hbar\kappa_0 c^2 - \left(1 + ie_2\right)\hbar\kappa_e c^2 = \hbar\kappa_p c^2 . \tag{3.52}$$

$$\hbar c\omega_0 - \Delta_{0-p}E = \hbar c\omega_0 - \hbar c\left(1 + ie_2\right)\delta\omega_0 = \hbar c\omega_0 - \left(1 + ie_2\right)\hbar c\omega_e = \hbar c\omega_p \tag{3.53}$$

Dividing through by the energy of the reflected wave, gives the coefficients of the driving, E_0, and resonant/transmitted, E_p, energies, in terms of their wave numbers and frequencies, and finally of rest masses with respect to the fundamental differentials

$$\frac{\kappa_0}{\kappa_e} - \left(1 + ie_2\right) = \frac{\kappa_p}{\kappa_e} \tag{3.54}$$

$$\frac{\omega_0}{\omega_e} - \left(1 + ie_2\right) = \frac{\omega_p}{\omega_e} \tag{3.55}$$

$$\frac{m_0}{m_e} - \left(1 + ie_2\right) = \frac{m_p}{m_e} \tag{3.56}$$

By virtue of (2.83) and the CODATA values for the ratio of the neutron-electron mass, (3.35), and (3.51) expressed in terms of a fundamental unit value of δ, we have

$$1838.6836598... - 2.5315844... = 1836.152075... . \qquad (3.57)$$

where the CODATA value for the last term is 1836.15267261(85). Expressing this ratio in terms of the fundamental resonant energy given by κ_p, ω_p, and m_p, we have

$$\frac{\kappa_e}{\kappa_p} = \frac{\omega_e}{\omega_p} = \frac{m_e}{m_p} = 0.000544617199... . \qquad (3.58)$$

The deviation from the CODATA value for (3.58), then is

$$0.000544617199... - 0.00054461702173(25) = 1.773... x10^{-10} . \qquad (3.59)$$

Alternately, working backwards from the CODATA value for (3.58) gives us a theoretical value of

$$\frac{\kappa_e}{\kappa_0} = \frac{\omega_e}{\omega_0} = \frac{m_e}{m_0} = 0.0005438671685... \qquad (3.60)$$

and a similar deviation from the CODATA value. The theoretically derived value of the neutron to electron energy ratio, then compared with (3.35) is

$$\frac{\kappa_0}{\kappa_e} = \frac{\omega_0}{\omega_e} = \frac{m_0}{m_e} = 1838.684256... . \qquad (3.61)$$

Cosmic Expansion Rate and Expansion Force

With respect to the expansion force, we would expect equating of the variance operator with respect to a change in the local scales, t_l and x_l, and the differential operator with respect to cosmic time, t, and space, x, so that

$$\delta\kappa_0 = \partial\kappa_0 = \kappa_e \text{ , and} \tag{3.62}$$

$$\delta\omega_0 = \partial\omega_0 = \omega_e \tag{3.63}$$

A change in the fundamental mechanical impedance, using (2.101) then becomes

$$\partial Z_0 = -\hbar\kappa_e\omega_e = -\hbar\kappa_e^2 c = -\frac{1}{c}\hbar\omega_e^2 \tag{3.64}$$

$$-\hbar\frac{\partial\kappa_0^2}{\partial t} = -\frac{1}{c}\hbar\frac{\partial\omega_0^2}{\partial x} \tag{3.65}$$

$$-\frac{\partial\lambda_0}{\partial t} = -\frac{1}{c}\frac{\partial\tau_0}{\partial x} = -\frac{\partial Z_0}{\partial x} \tag{3.66}$$

In this final arrangement, a change in the fundamental linear inertial density over time arising from expansion is viewed as a conserved or unitary mass/energy distributed over an increasingly larger unit length, and is equal to the change in continuum impedance over a changing unit distance and scale. This can be seen graphically by referring to the Matrix of Invariants. As the inertial density differential is a linear function of that changing unit length, and as the time and distance scales are held to be invariant with respect to each other by virtue of c, then the time scale is increasing as well. In terms of a fixed scale of time, however, present, past or future, it is apparently an exponential function and in fact a measure of the cosmic expansion rate, X_e, and we should find

$$X_e = -\frac{1}{c}\hbar\frac{\omega_e^2}{\partial x} = -\left|\frac{1}{c^2}\hbar\omega_e^2\right| = -\left|\frac{1}{c^2}\partial\tau_0\right| = -\hbar\frac{\kappa_e^2}{\partial t} = -\frac{\partial\lambda_0}{\partial t} \text{ .} \tag{3.67}$$

where the figures in absolute value brackets are converted from the second term by complex integration with respect to time.

Substituting the product of (3.60) and the CODATA based value of ω_0, gives an evaluation of

$$X_e = 2.35896879...x10^{-18}\,\Delta m/m/s \tag{3.68}$$

where the accelerating change in wave number is masked by the change in unit length.

This number, the rate of change in a meter unit of spacetime, per second, times the number of meters per megaparsec, gives the expansion rate in terms of the Hubble constant or

$$H_0 = X_e\left(3.08572x10^{22}\,m/mps\right) = 72{,}791.17172\,m/mps/s \tag{3.69}$$

A study by Ron Eastman, Brian Schmidt and Robert Kirshner in 1994 and quoted in Kirshner's recent book, The Extravagant Universe, found an H_0 = 73 km/s/mps +/- 8km and an article in the *Astrophysical Journal*, 533, 47 - 72, (2001) by Freedman, W. L. et al.

gives the final results from the Hubble space telescope key project to measure the Hubble constant as H_0 = 72 km/s/mps. There are $3.08572x10^{22}$ meters per megaparsec.

Thus, if the Hubble rate of expansion is roughly 73 kilometers per second per megaparsec, and since there is no logical compulsion to think that we are at the current center of the universe, (except in the sense of a 3-space layer moving out from the center of a 4-core), this would tend to indicate that every local section of space, absent gravitational and electromagnetic constraints, is moving away from every other at approximately 2.35 $x10^{-18}$ meters per second per meter of separation. It follows logically that inversion of this number will give us the approximate time since all the matter was at the same locale, that the universe has been expanding, or 4.25 $x10^{17}$ seconds, which is roughly 13.5 billion years.

However, as this number might be deemed to represent an expansion via a compounded augmentation of the scale of spacetime itself, and not simply an extension of matter within that spacetime, we might surmise that this represents an exponential expansion, in which case the following equation for the doubling of spacetime applies, as

$$\tau_H = \mathbf{ln}\, 2 X_e^{-1} = 2\mathbf{.}938x10^{17}\, s = 9\mathbf{.}311 \text{ billion years} \; , \qquad (3.70)$$

This indicates that spacetime is doubling at a current rate of every 9.311 billion years, measured in terms of today's seconds. If we assume that the wavelength of the cosmic background radiation at approximately 5mm embodies that augmentation, while harkening back to a period of primal beta decay as indicated by the Compton wavelength over 2π of an electron, this represents a doubling of some 30 times, or

$$\frac{\mathbf{ln}\left(\dfrac{\frac{.005}{2\pi}}{r_e}\right)}{\mathbf{ln}\, 2} = \frac{\mathbf{ln}\, 2\mathbf{.}060\mathbf{...}x10^9}{\mathbf{ln}\, 2} = 30\mathbf{.}94\mathbf{...}doublings \qquad (3.71)$$

a lifetime in terms of today's measure of time of roughly 288 billion years.

As

$$\mathbf{ln}\, 2 = 0\mathbf{.}693147181\mathbf{...} \qquad (3.72)$$

it is worth noting that this figure is effectively 70%, the factor of expansion attributed in current cosmological schemes to dark energy.

As this expansion rate as found in the Hubble constant is also found in the differential change in the linear inertial density and tension force operating on a fundamental quantum oscillation, it is confirmation of the fact that cosmic expansion provides the emf driving such oscillations.

Returning to (2.144), rearrangement gives

$$\partial \text{ת}_2 \equiv \text{ת}\omega_e^2 = \text{ת}\partial\omega_0^2 = \frac{k^2 i^2}{\alpha'} = 0\mathbf{.}212013542\mathbf{...}N \; . \qquad (3.73)$$

Returning to (2.146), which we can rearrange and solve, using the CODATA value for elementary charge, gives

$$ki = e\partial\omega_0 = e\omega_e = 124\mathbf{.}3839844\mathbf{...}\, i \; (amperes\mathbf{)} \; . \qquad (3.74)$$

Thus the SI current per quantum differential change in frequency associated with beta decay is equal to the fundamental charge of the system. If we return to (2.147) and divide through by *k* to arrive at a normalized and naturalized value of elementary charge, e', we have,

$$\frac{e}{k} = e' = \frac{\partial i}{\partial \omega_0} = \frac{1}{\omega_e} = 1\mathbf{.}288089085\mathbf{...}x10^{-21}\, s/\theta \tag{3.75}$$

which shows charge to be the inverse of the quantum differential change in angular frequency.

A final speculation regarding beta decay concerns the decay rate of the neutron, τ_n, which is reported in a paper by R.R.Kinsey, et al.,*The NUDAT/PCNUDAT Program for Nuclear Data* as being 624 seconds. Other recent sources report it in the 887 +/- 2 second range. If instead of the standard use of the *ln* 2 as the dividend used in computing a half life, in light of the above analysis of a 4-wave, we use the *ln* $\sqrt{2}$ to indicate the doubling of a 4-core, we have

$$\tau_n = \frac{\mathbf{ln}\sqrt{2}}{\frac{\omega_e}{\omega_0}} = 637\mathbf{.}239\mathbf{...} \text{ seconds} . \tag{3.76}$$

4 – Strong Interactions

The above development shows the generation of quantum gravity and the electroweak interaction as a function of the spin dynamics of a fundamental oscillation. As functions of a single quantum, they are not so much "interactions" as intra-actions within the domain of a discrete, rotational oscillation of the spacetime continuum. With respect to interactions <u>between</u> separate oscillations, these functions are quantized by the fundamental characteristic dynamics of that continuum. The strong "force" that binds nucleons, by contrast, is truly a function between and among separate quanta, and can be understood to vary continually within a certain range.

According to Boyle's Law, given adiabatic constraints, the energy per volume of a gas is equal to the pressure on the boundary of that volume. If we apply a similar logic to our discussion of spacetime in which expansion appears to be adiabatic, then density of the spin energy, E_0, within the boundary of a rotational oscillation will be equal to the tension at that boundary, held to be generally spherical, as

$$\frac{E_0}{V} = \frac{\tau_0}{\mathrm{A}_0} = f_0 \text{ where} \tag{4.1}$$

$$\frac{E_0}{V} = \frac{i\hbar\kappa_0^{-1}\omega_0^2}{\frac{4}{3}\pi\left(i\kappa\right)_0^{-3}} = -\frac{3\hbar\kappa_0^2\omega_0^2}{4\pi} \tag{4.2}$$

The tension, however, is also a function of the boundary configuration, as a given volume can be bound by a variable surface area, so that a decrease in boundary area, given no change in volume, must result in a decrease in the wave force if the energy is to be conserved. Disregarding the small difference between the spin energy of the neutron and proton, if two equal volumed quanta with the same energy density are brought in synchronic contact so that there is <u>no</u> tension <u>gradient</u> at their common boundary, the decrease in their total boundary area <u>with</u> <u>gradient</u> before and after conjunction results in a proportional decrease in the wave force of each. We can approximate this in general terms by looking at the difference between the combined surface area of two separate, equal spherical volumes and the surface area formed by two hemispheres connected by a cylinder of like radius about their combined volume. The following table shows this as,

Wave State	Volume	Total Surface Area	Difference	% Difference
2 Single Waves	$\frac{8\pi}{3}r^3$	$8\pi r^2$		
2 Conjoined Waves	$\frac{8\pi}{3}r^3$	$4\pi r^2 + \frac{8}{3}\pi r^2$	$\frac{4}{3}\pi r^2$	1/3 of a Single 1/6 of Combined

We might expect, then, a reduction in wave force of 1/6 to 1/3 of the quantum fundamental with the addition of each nucleon to an atomic nuclear congregation, which would result in an apparent reduction by a related amount in the mass/energy of the system. Hence this energy appears to be bound up in the nucleus, with the actual amount depending on the geometric configuration of the nucleus. In effect what occurs is a conversion of spin energy density, E_1, to the potential energy density of continuum mass, V_1, as a reduction in the wave number, and a net decrease in the wave tension, f_0, where

$$-\Delta E_1 = +\Delta V_1 \text{, where} \tag{4.3}$$

$$E_1 = \frac{3f_0}{4\pi} = \frac{3\tau_0}{4\pi \mathrm{A}_0} = \frac{3\tau_0}{4\pi}\kappa_0^2 \tag{4.4}$$

In light of the above development of the nucleons as strain oscillations of spacetime, this shows the energy-stress force, E_0-3τ_0, to be divided over the 4π steradians of the spherical volume. Therefore, the energy density of (4.4) is the energy per steradian of the oscillation. Taking the square root gives us the energy, and according to the matrix of invariants the other dynamic functions of spin, per radian of strain travel. Thus the ranges from the table above give the change of (4.3) per radian and relate that change to the primary parameters, ω_0 and κ_0, and thereby to energy and mass, and we have

$$\sqrt{\frac{1}{6}E_1} < \Delta E_1 < \sqrt{\frac{1}{3}E_1}\text{ ,} \tag{4.5}$$

$$\sqrt{\frac{1}{6}\frac{3}{4\pi}f_0} < \Delta f_0 < \sqrt{\frac{1}{3}\frac{3}{4\pi}f_0}\text{ .} \tag{4.6}$$

This change in energy or binging energy is generally measured in million electron volts, Mev. In the Mev system, mass is converted to elementary charge and the time and distance dimensions are set to 1 or 1 $Mev = \left(mc^2/e\right)x10^6$. The binding energy per nucleon observed is in the 6 to 8.794… Mev range for all but the lightest elements, for which it is less. The upper end is the value for the binding energy of nickel, ^{62}Ni, as the most stable of elements. The only stable elements beneath the lower threshold are hydrogen, helium-3, and lithium, with helium-4 in-range at a stable 7.074 Mev.

The mass of the neutron is then 939.565…Mev, and the proton is 938.272…Mev. The energy density per steradian for their average is then

$$\frac{m_0c^2}{eV} = \frac{3\left(938.918\ldots\right)}{4\pi} = 224.150\ldots Mev \tag{4.7}$$

The range for the change in energy due to nuclear congregation as derived above is then

$$\begin{gathered}\sqrt{\frac{224.150\ldots}{6}}Mev < \Delta E_0 < \sqrt{\frac{224.150\ldots}{3}}Mev \\ \left(6.112\ldots < \Delta E_0 < 8.643\ldots\right)Mev\end{gathered} \tag{4.8}$$

Note that the square root operation is effectively the reverse of a cross product, in which the crossing of two orthogonal tangent vectors produces a radial or normal vectors. In that case the crossing of two forces can result in a third orthogonal force. Here, the square root decomposes a normal vector into two orthogonal tangent vectors. The resultant energy or force vectors, depending on the particular dynamics being analyzed, are in units of energy or force. The square root operates on Mev and not ev, since the basic units of energy density and stress are volume units and the unit Mev is in the same scale as the energy of the electron at .510… Mev or the difference between the proton and neutron at 1.293…Mev. Thus the SI stress force equivalent to energy in Mev remains in Newton or mass distance per time squared, and not the square root thereof.

As a refinement, with respect to nuclear congregation, twelve equal volume spheres will pack tightly around a thirteenth, so that we might imagine instead of distribution of the energy over 4π steradians, a maximum distribution over 12 equal sectors of $\pi/_3$ steradians. The resulting range then becomes

$$\sqrt{\frac{234.729\ldots}{6}}Mev < \Delta E_0 < \sqrt{\frac{234.729\ldots}{3}}Mev, \qquad (4.9)$$
$$\left(6.254\ldots < \Delta E_0 < 8.845\ldots\right) Mev$$

the upper boundary of which is slightly over the observed upper limit to binding energy per nucleon. This is the energy per $\sqrt{\frac{\pi}{3}}$ radians. Note the tie in of this configuration with the comments on the weak mixing angle at (2.58).The top four elements in this regards, two nickel and two iron isotopes, taken from a study by Wapstra and Bos quoted at http://hyperphysics.phy-astr.gsu.edu/hbase/nucene/nucbin2.html, are

Nuclide	Mev per nucleon
^{62}Ni	8.79460 +/- 0.00003
^{58}Fe	8.79223 +/- 0.00003
^{56}Fe	8.79036 +/- 0.00003
^{60}Ni	8.78079 +/- 0.00003

Finally, with respect to the strong interaction and the quantum gravity differential, using the upper limit for comparison, we have a theoretical dimensionless ratio between the two

$$\frac{F_S}{F_{Gq}} = \frac{\sqrt{\frac{\pi}{3}}\sqrt{\frac{1}{4\pi}}\tau_0}{G_q dT} = \frac{\sqrt{\frac{1}{12}}\hbar\omega_0^2}{\frac{1}{6\sqrt{3}\kappa_0^2}dT} = \frac{3\hbar\kappa_0^2\omega_0^2}{dT} = 4.8750635\ldots x10^{37}\, T\!\Big/\!dT \qquad (4.10)$$

Comparing this with (4.2), we see that it is equal to the spin energy-stress density of the oscillation integrated over 4π steradians; that is, the volume with respect to E_0 and the surface area with respect to f_0.

5 - Cosmological Considerations and Speculations

Whether we see the decrease in inertial density with expansion as an elastic strain and concomitant stress pulling on the rotating wave front of a quantum oscillation or the densification of that front as an inertial body force pushing back on that front, the existence of forces that result in the restoration of any strain of the wave medium defines it as being elastic. Even an instance of inertial wave motion, as with an ideal jump rope, in which the kinetic energy is constrained, relies on the modulus of rigidity of the rope for the inertial constraint. As the modulus of rigidity is the shear modulus which can be expressed as a function of some tension modulus, it is clear that even an example of an ideal inertial wave depends on elastic properties for its expression. Attempting to make a jump rope out of bread dough will prove the point after only a few rotations. Apparently then, inertia and elasticity, or inertial density and tension/shear stress-strain are two phenomenalogical aspects of an underlying inertial continuum ontology.

Perhaps the real question is whether or not, in addition to being elastic, the spacetime continuum is capable of plasticity. Obviously, the elastic limits cannot be exceeded in the short term on the level of a quantum wave, or the wave would quickly dissipate, i.e. the medium would not sustain oscillation. Attempt to make a drumhead out of uncooked bread dough. On the larger scope of things, the vast areas of cosmic space devoid of galactic presence suggest regions of spacetime that have exceeded the limits of oscillatory strain and are in plastic flow. The whole issue of the missing dark matter which is deemed to hold sway at the peripheries of galaxies might itself vanish if we find that the tug between the galactic nuclear dynamics of gravity with increasing angular momentum and the isotropic expansion of the galactic environment are sufficient to exceed those elastic limits in the middle interstellar regions, leaving the outer regions to follow the dictates of angular momentum while the center region is under the domain of gravity and angular acceleration.

A terrestrial analogy of sorts can be found in the demolition of an old industrial brick chimney. When of sufficient height and toppled by explosion so that it pivots at its base, the potential for angular acceleration of the top can exceed that of gravitational interaction or free fall alone. If the shearing force arising from angular acceleration exceeds the bonding strength of the mortar, that acceleration can not be transmitted to the top portion of the chimney, which will break into two parts. The topmost part of the bottom portion is accelerated by angular forces greater than the gravitational ones and falls first with greatest velocity. The speed at impact for any differential length of the bottom section is proportional to its distance from the center of rotation, while the top portion is under the sway of gravity alone and hits the ground last, all portions at the same velocity.

The spacetime continuum itself is the cosmic mortar that keeps the quantum waves that comprise the stellar systems together. In a somewhat different scenario from our falling chimney, if a region of spacetime itself is under rotational stress to convey a common angular acceleration to the halo around a galactic core, we might imagine the rigidity of spacetime to be maintained to a yield boundary after which the shearing stresses would exceed the limit imposed by the shear modulus and a plastic state would ensue. Barred spiral galaxies would indicate an inner region of relative rigidity and elastic integrity of spacetime, with conventional spirals indicating a wider area of plasticity. As the tangential component of the angular acceleration increases, so does the centripetal component, and as the stress at the outer extents of the galactic arms increases to the yield limit, a state of plasticity ensues between those extents and the adjacent inner section. The new outer extent of the area within the galactic elastic circumference next accelerates to the tangential velocity productive of plastic flow, resulting in a differential shrinkage of the elastic circumference. The angular momentum already imparted to the outer regions at the inception of the yield would be maintained as a constant tangential velocity.

Apparently regions of maintained elasticity can exist within larger regions of plasticity and vice versa. If the elasticity, as the stress-strain relationship, is a function of inertial density, it would appear to be independent of time, and therefore reversible. Any geometrically defined process resulting in a densification or confinement of a plastic region would be capable of restoring elasticity.

With that caveat, we can now join the pieces of the spin function with the expansion stress. For an elastic medium of tension stress T_0 , (where we call the stress a tension stress, although it is orthogonal to all of 3-space), with a strain of $\varepsilon = \frac{\Delta l}{l} = \frac{r_0}{1\,meter} = \kappa_0^{-1}$, the orthogonal modulus of elasticity, Y, is

$$\begin{aligned} Y &= i\frac{T_0}{\varepsilon} = i\frac{T_0}{r_0} = 6\sqrt{3}\left(i\kappa_0\right) f_0 = 6\sqrt{3}\,{}_3\text{ת}_2 \\ &= i\frac{1.68877...x10^{38}\,N/m^2}{2.10019...x10^{-16}\,\Delta m/m} = i8.041025...x10^{53}\,N/m^2 \end{aligned} \tag{5.1}$$

The 4-D, either hyper-volume or spacetime, potential force density then is equal to the inverse of the Planck area, A*Planck*, keeping in mind that for the latter, the units shown are of the derivative of area with respect to stress and not the differential area, *d*A, alone

$$\begin{aligned} F_1 &= -\frac{Y}{r_0} = \frac{-T_0}{\varepsilon^2} = 6\sqrt{3}\left(i\kappa_0\right)^2 f_0 = \frac{f_0}{G_q} = 6\sqrt{3}\,{}_4\text{ת}_2 \quad \text{and} \\ &= -\frac{1.68877...x10^{38}}{4.41082...x10^{-32}} = -3.82871...x10^{69}\left(N/m^2\right)\Big/m^2 = -\mathrm{A}_{Planck}^{-1} \end{aligned} \tag{5.2}$$

$$\mathrm{A}_{Planck} \equiv \frac{d\mathrm{A}_0}{dT_0} = \frac{\mathrm{A}_0}{T_0} = \frac{2.61184...x10^{-70}\,m^2}{1N/m^2} \tag{5.3}$$

The inertial notation, in which all orthogonal sense are tacit, points to the fact that although the modulus of elasticity is conventionally given the same dimensions as the stress, and the strain is a dimensionless number, the modulus in this case is actually an orthogonal stress derivative with respect to a change in extension or strain, which is a linear-stress/volume-force potential, as

$$f_0 = Yi\Delta r = \frac{1}{6\sqrt{3}}\frac{dT_0}{idr}i\Delta r = \frac{d\tau_0}{d\mathrm{A}\cdot idr}i\Delta r = \frac{dE_0}{d\mathrm{A}\cdot\left(idr\right)^2}\left(i\Delta r\right)^2 \tag{5.4}$$

which upon being integrated by iΔr, (dividing by iκ in the Euler formalism), produces a stress, f, and (5.2) is then a 4-D force potential, which is integrated by the quantum gravitational constant, so that

$$f_0 = F_1 G_q = \frac{G_q}{\mathrm{A}_{Planck}} \tag{5.5}$$

With respect to the rest of the parameters of the Planck scale, the square root of (5.2) gives us the inverse Planck length or what we can call the Planck wave number as

$$\mathrm{A}_{Planck}^{-\frac{1}{2}} \equiv r_{Planck}^{-1} \equiv \kappa_{Planck} = 6.18765\mathbf{...}x10^{34}\,m^{-1} \tag{5.6}$$

which with (2.84) gives the Planck mass, m_{Planck},

$$m_{Planck} \equiv \text{ת}\left(i\kappa_{Planck}\right) = 2.17661\mathbf{...}x10^{-8}\,kg \tag{5.7}$$

and with c gives us the inverse Planck time, or what we can call the Planck frequency as

$$t_{Planck}^{-1} \equiv \omega_{Planck} \equiv c\left(i\kappa_{Planck}\right) = 1.85501\mathbf{...}x10^{43}\,s^{-1}\,. \tag{5.8}$$

Inverting (5.6) and (5.8), to express them as derivatives as in (5.3),

$$x_{Planck} \equiv \frac{d\,\mathrm{A}_{Planck}^{\frac{1}{2}}}{dT_0} = \left|\frac{x_0}{\sqrt{T_0}}\right| = \left|\frac{x_0\sqrt{T_0}}{T_0}\right| = \frac{1.61612\mathbf{...}x10^{-35}\,m}{1N\,/\,m^2} \tag{5.9}$$

$$t_{Planck} \equiv \frac{c^{-1}dx_0}{dT_0} = \frac{dt_0}{dT_0} = \left|\frac{c^{-1}x_0}{\sqrt{T_0}}\right| = \left|\frac{c^{-1}x_0\sqrt{T_0}}{T_0}\right| = \frac{5.39080\mathbf{...}x10^{-44}\,s}{1N\,/\,m^2} \tag{5.10}$$

and doing the same with (5.7), we have

$$m_{Planck} \equiv \frac{\text{ת}\left(i\kappa_{Planck}\right)}{dT_0} = \left|m_0\sqrt{T}\right| = \left|\frac{m_0 T_0^{\frac{3}{2}}}{T_0}\right| = \frac{2.17661\mathbf{...}x10^{-8}\,kg}{1N\,/\,m^2}\,. \tag{5.11}$$

In terms of a present local section of spacetime, (5.3), (5.9), and (5.10) simply express relationships of the familiar invariants at those local conditions in natural units, as can be seen in the next to the last term of each, followed by the SI equivalents. It is the large magnitude of T_0 in SI units that makes the scale so small, which it is in terms of local natural units, x_0. If the value of x_{Planck} is extrapolated back to an initial condition of unity at the primal emanation, and T_0 retains its current SI value, then x_0 is expressed in units of x_{Planck} and is a measure of the extent of expansion of the cosmos, assuming maximum possible density at the primal emanation.

We can make one adjustment to these identities that will facilitate the final development by normalizing the value of c. We can do this by using the light second, l_{ls}, the distance light travels in one second, as our unit of length, so that the Planck length becomes

$$x_{Planck} \equiv \frac{d\,\mathrm{A}_{Planck}^{\frac{1}{2}}}{dT_0} = \left|\frac{x_0\sqrt{T_0}}{T_0}\right| = \frac{5.39080\mathbf{...}x10^{-44}\,l_{ls}}{1N\,/\,m^2} \tag{5.12}$$

Conversely, we can use the light meter, t_{lm}, the time it takes light to travel one meter, as our unit of time. This is within an order of magnitude of one nanosecond, in which case,

$$t_{Planck} \equiv \frac{c^{-1}dx_0}{dT_0} = \frac{dt_0}{dT_0} = \left|\frac{c^{-1}x_0\sqrt{T_0}}{T_0}\right| = \frac{1.61612\mathbf{...}x10^{-35}\,t_{lm}}{1N\,/\,m^2}\,. \tag{5.13}$$

It must be understood, however, that if we do this for the Planck scale, we must do it for our current local scale. Thus the angular wave number in terms of the light second is

$$\kappa_0 = 1.42745\ldots x10^{24}\,\theta/l_{ls} \tag{5.14}$$

and the angular frequency in terms of the light meter is

$$\omega_0 = 4.76146\ldots x10^{15}\,\theta/t_{lm} \tag{5.15}$$

We also make the following geometric observation with respect to (5.4). An increase in the radius of a sphere or torus or a normal radial of a cube, results in an increase by the same proportion squared to the surface area, subject to geometric configuration. Therefore, normally we would look for a change in the inertial density, λ_0, to be a linear function of ∂x through a change in the wave number κ_0, with a change in T_0 as a function of ∂A and a squaring of ∂x. Thus, given the following, where the bracketed figure in the second term appears to be the oscillatory spring constant,

$$\lambda_0\omega_0^2 = -i\kappa_0\left(-m_0\omega_0^2\right) = -i\kappa_0 k_{spring} = \text{ת}\kappa_0^2\omega_0^2 \equiv \frac{\tau_0}{\mathrm{A}_0} = f_0 \tag{5.16}$$

differentiating for the end terms, we have

$$\frac{\partial\lambda_0}{\partial\kappa_0} = \kappa_0^2\left(k_{spring}\right) \tag{5.17}$$

$$\frac{\partial f_0}{\partial\mathrm{A}_0} = -\frac{\tau_0}{\mathrm{A}_0^2} = -\frac{f_0}{\mathrm{A}_0} \tag{5.18}$$

$$\frac{\partial\lambda_0}{\partial\kappa_0} = \kappa_0^2\left(k_{spring}\right) = \mathrm{A}_{Planck}^{-1} = -\frac{6\sqrt{3}f_0}{\mathrm{A}_0} = \frac{\partial T_0}{\partial\mathrm{A}_0} \tag{5.19}$$

indicating that the inverse Planck area divided by the wave number squared is the oscillatory spring constant of spacetime. The problem with this is that the equality is valid only if

$$k_{spring} = -6\sqrt{3}m_{\partial\kappa}\omega_0^2 \text{ where } m_{\partial\kappa} = m_0\kappa_0 \tag{5.20}$$

so that

$$k_{spring} = -6\sqrt{3}\text{ת}\kappa_0^2\omega_0^2 = -T_0 \tag{5.21}$$

showing that the linear inertial density varies with the square of the augmentation of the length scale, in keeping with the identity, $\lambda_0 = \text{ת}\kappa_0^2$, and points to its invariance vis a vis the other wave functions. Thus we are back to (5.16), which can be restated as

$$\frac{\partial\lambda_0}{\partial t_0^2} = \frac{\partial\tau_0}{\partial x_0^2} = f_0 \tag{5.22}$$

and see that a change in inertial density with a stretching of the length scale is an accelerating change equal to the change in force per change in area or dynamic stress, which we might find is exponential. We can think of this in physical terms as the constant prevailing over inertia of the expansion force.

Returning to (5.12) and (5.13), a change in the scale of x or t is a function of the square root of the change in stress, an expression of the field strength, and for a normalized c is

$$x_{Planck} = \mathrm{A}_{Planck}^{\frac{1}{2}} = \frac{x_0}{i\sqrt{T_0}} = \frac{1}{i\kappa_0\sqrt{k_{spring}}} = \frac{x_0}{i\sqrt{k_{spring}}} \tag{5.23}$$

$$t_{Planck} = \mathrm{A}_{Planck}^{\frac{1}{2}} = \frac{t_0}{i\sqrt{T_0}} = \frac{1}{i\omega_0\sqrt{k_{spring}}} = \frac{t_0}{i\sqrt{k_{spring}}} \tag{5.24}$$

Inverting gives

$$\kappa_{Planck} = \mathrm{A}_{Planck}^{-\frac{1}{2}} = i\kappa_0\sqrt{T_0} = i\kappa_0\sqrt{k_{spring}} \tag{5.25}$$

$$\omega_{Planck} = \mathrm{A}_{Planck}^{-\frac{1}{2}} = i\omega_0\sqrt{T_0} = i\omega_0\sqrt{k_{spring}} \tag{5.26}$$

and mass is simply the product of κ_{Planck} and the inertial constant.

The inverse Planck parameters, (5.2), (5.6), and (5.8) and the mass term, (5.7), only have significance if we image that at some initial condition at primal emanation, but not necessarily a hot, big bang, they were all at a general condition of unity in the SI system. The wave number in (5.6) is in terms not of a meter now, but of the universal whole now or of a cosmic unity or extent, C_x. Thus we imagine the whole of the universe, at some primal epoch, collapsed to a sphere or horn torus of maximum density, in which the radii of all oscillations would ideally be in contact. If we think of the scale of those radii as being in terms of the current interpretation of the Planck length, then the scale of C_x would be one meter. If we think of that scale as being the current scale of x_0, at 10^{-16} meters, then C_x would be in the general neighborhood of 10^{11} meters, assuming a population of 10^{80} nucleon. Comparing this value with the current value for κ_0 we have the exponential factor for the change in κ_0 due to a change in inertial density from expansion

$$\kappa_{\mathbf{exp}} = \frac{\kappa_{Planck}}{\kappa_0} = \sqrt{T_0} = \frac{6.18765\ldots x10^{34}\ \theta/R_C}{4.76146\ldots x10^{15}\ \theta/m} = 1.29952\ldots x10^{19} \tag{5.27}$$

indicating that the measure of unity has decreased, vis a vis the whole, according to the square root of the tension, T_0, from a universal unit, by the inverse factor of $7.69510\ldots x10^{-20}$ to a current meter.

In similar fashion to (3.70) we can get a figure for the extent of cosmic expansion through a doubling of that extent, in terms of light seconds as

$$C_x = \mathbf{ln}\,2\left(\kappa_{\mathbf{exp}}\right) = 9.00764\ldots x10^{18}\,s \tag{5.28}$$

where we note the tie-in of this logarithmic change in the field with the discussion on beta decay at (3.42) and in the appendix on exponentiation at (7.139). This extent divided by the doubling rate in (3.70) gives us the number of times that extent has doubled or

$$\frac{C_x}{\tau_H} = \frac{9.00764\ldots x10^{18}}{2.93834\ldots x10^{17}} = 30.655\ldots \tag{5.29}$$

Approached another way, dividing the value of the expansion rate in (3.68) by the inverse of (5.27), where the expansion rate is the rate of decrease in the inertial density and thereby a measure of the linear scale, gives

$$\frac{X_e}{\kappa_{\mathbf{exp}}^{-1}} = \frac{2.35896...x10^{-18}}{7.69510...x10^{-20}} = 30.655... \tag{5.30}$$

for a factor of a little over thirty times the increase from x_0 to 1 meter and an age of the cosmos of approximately 285 billion years. This compares very closely with (3.71) at 30.94 and 288 billion years which is based on the background microwave wavelength, and supports the assertion that the expansion of the cosmos is exponential and structured according to this development.

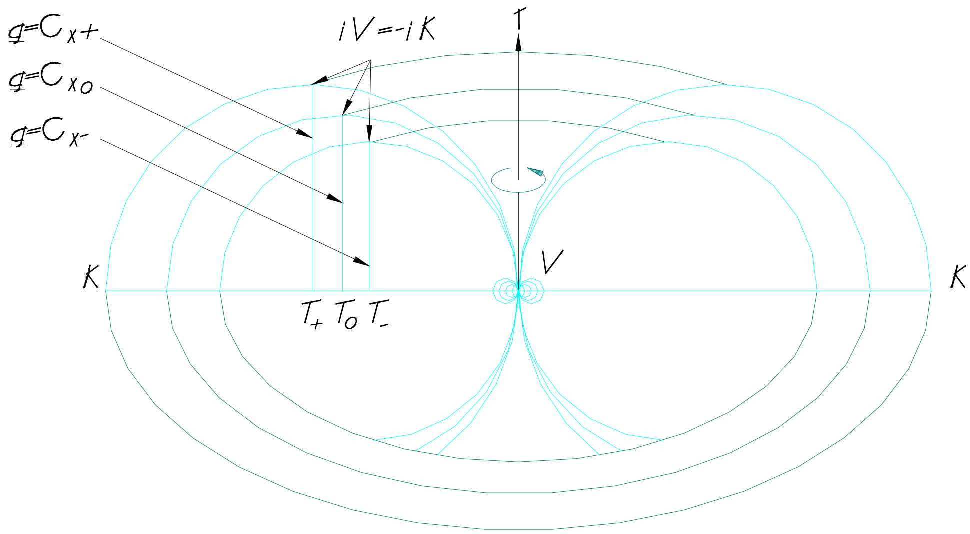
With respect to the Planck frequency, ω_{Planck}, the unit of time is once more universal and is in fact the time lapsed since the universal expansion began,

$$\omega_{\mathbf{exp}} = \frac{\omega_{Planck}}{\omega_0} = \sqrt{T_0} = \frac{1.85501...x10^{43}\ \theta/T_C}{1.42745...x10^{24}\ \theta/s} = 1.29952...x10^{19} \tag{5.31}$$

What this says, of course, is that a second today is not what it was 285 billion years ago, but then again, neither is a meter, so c remains invariant.

The Planck mass, then is the product of (5.27) and m_0, the neutron mass, and gives its value at that early epoch, m_I. The remainder of the early epoch values for oscillation functions, as in the orthogonal matrix of invariants, can be found by substituting the values of ω_{Planck} and κ_{Planck} for the current values. At this scale the spin energy of a neutron would be, 1.95624…x10^9 joules, the Planck energy. Based on this line of thinking, the potential energy density as in (4.4) at that epoch would be 1.10638…$x10^{113}$ joules/cubic meter or 1.23101…x10^{96} kg/cubic meter. This assumes that the wave number is still the measure of mass and that the wave number of (5.6) is per today's meter, so that each oscillator is confined to a volume of roughly 10^{-105} cubic meters. As there are estimated to be roughly 10^{80} hadrons in the known universe, this indicates that at this scale the entire known compliment of matter would fit in a volume of approximately 10^{-25} cubic meters or roughly one cubic nanometer. The inertial density would be the same, regardless of whether there where any actual oscillations at that time, and though of great magnitude, is hardly infinite and does not suggest a singularity. The stress T_0 would be 4.81635…x10^{114} Newton/meter squared. The transverse wave force would be 1.21043…x10^{44} Newton.

If we idealize the cosmic structure as a horn torus, in which the center hole consists of a dense inertial locus, instead of as a sphere, then the horn becomes the center of the primal emanation or "big bang". Functional continuity is thereby maintained at and through the center, and expansion of a 3-D space from maximum density at that center and out around the annulus, with rotation about the axis, will lead to decreasing density with that strain, until such time as it reaches a half revolution of the annulus. From that locus the return toward the center will increase the density of the continuum as it is constrained back toward the horn in a cosmic inertial sink or "black hole". A similar effect would be produced by an oscillation through the horn, without the circumnavigation of the annulus, with some modification. This is more in keeping with the dictates of continuum logic, as circumnavigation of the annulus would appear to assume some type of ideal fluid, which has not been a part of our assumptions, i.e. no point neighborhoods that can flow past each other. Only stresses are assumed to be able to freely rotate. Finally, the structure might echo that of the fundamental quantum spin diagrams, expanding and contracting over cosmic time.



2-D Representatioon of a 3-Torus

With conceptual respect to the principles of expansion, the center of the torus comprises a locus, *V*, of maximum potential energy, and the extent of the annulus in the plane of the torus comprises a circumference, *K*, of maximum kinetic energy. This gives a linear potential energy density of

$$V_1 = \lambda_V c^2 = \tau_V \tag{5.32}$$

and a kinetic energy density of

$$K_1 = -\lambda_K c^2 = \tau_K\,, \tag{5.33}$$

both of which are equal to a tension force, τ, so that

$$V_1 = -K_1 g' \tag{5.34}$$

The negative sense reflects the relationship of Wave Diagram 2, in that the velocity/momentum and displacement are always of opposite sense. We might anticipate that τ_K vanishes at the extremum, since it represents a point of directional change, maximum momentum and zero acceleration, but this is so with respect to the central toric plane only, as it still has a centripetal component of angular acceleration about the polar axis. The horn torus represents an inversion of sorts of Wave Diagram 1, in which the antinodes of the central rotating *x* axis of that spin map to the toric central plane and the path of the nodes in the *Y-Z* plane map to the toric center. It is symmetrical to Spin Diagram 1 with the *V* poles brought together at the center of the sphere. The g' is a geometric factor that maps V_1 onto K_1. The points of maximum instantaneous power of the wave are found at *iV* = -*i K* and the flat space of the torus.

The sum of the power at the locus of points *K* should be equal in magnitude to the power at point *V*, which we can express as

$$P = cE_1 = cV_1 = -c\sum K_1 \tag{5.35}$$

Assuming the invariance of c, combined with (5.32) & (5.33), this can be reduced to a contrast of the inertial densities, or

$$\hbar\kappa_V^2 = -\hbar\kappa_K^2 g' \text{, and} \tag{5.36}$$

$$g' = 2\pi\left(2\underline{g}\right) = -\frac{\kappa_V^2}{\kappa_K^2} \tag{5.37}$$

We might anticipate that $\underline{g}$ is a 4-vector representing the radius of the toric annulus, in which $\sqrt{3}x_0$ is a 3-vector, C_x is the cosmic expansion extent, a 4-vector, equal to $\underline{g}$ and to T, the cosmic time elapsed since a locus of stress at V has transformed to a point on the plane of the upper annulus extremus at, $iV = -iK$. Thus

$$\underline{g} = \underline{C}_x = \sqrt{x_{Planck}^2 + \left(\sqrt{3}x_0\right)^2} = \mathbf{ln}\,2\left|\kappa_{\mathbf{exp}}\right| = \mathbf{ln}\,2\left|\sqrt{T_0}\right| \tag{5.38}$$

where the Planck length is an invariant equal to the initial $x_I = x_0$ at $T = 0$

$$x_{Planck} = x_I = \sqrt{\underline{C}_x^2 - \left(\sqrt{3}x_0\right)^2} \tag{5.39}$$

A cosmic time vector, **T**, can be envisioned through the horn and in the case of the revolution of the annulus, remains locally tangential to the annulus as T. We might further imagine that **T** is an axial vector, imparting rotation to the torus as a whole. In the case of an oscillating structure, the vector simply reverses with the oscillation. We can assign a spatial axis and vector, **X**, which is parallel to $\mathbf{T}_0$ at time zero at the center of the horn. It is immaterial which vector is stationary and which follows the annular tangent. Thus T constitutes a fourth spatial vector of motion, which we might call W, and which is locally indistinguishable from X, Y and Z, but which is normal to all three, as indicated previously. T is not to be confused with the t dimension of a standard model four-vector, as we are here interested in a time whose unit scale does not change with a change in spatial scale with expansion, and whose product with a cosmic frequency, Ω, will range between 0 and ½ π.

Any three-dimensional locus of points emerging from the horn undergoes an expansion which is perceived locally as isotropic. While the isotropic nature of the points equidistance from the horn center is apparent, the matter of those closer and further from that center require some scrutiny. The points at T_+ have a decreased inertial density from those at T_0, which is itself decreased from T_-, which indicates a gradient and an anisotropic condition. In addition, there would be no net motion of the locus from the center unless the expansion force, τ, bore the following relationship to T, or

$$\tau_{T_-} > \tau_{T_0} > \tau_{T_+} \tag{5.40}$$

as with the inertial densities

$$\rho_{T_-} > \rho_{T_0} > \rho_{T_+} \text{ and} \tag{5.41}$$

$$\lambda_{T_-} > \lambda_{T_0} > \lambda_{T_+} \tag{5.42}$$

and with the expansion stress, f,

$$f_{T_-} > f_{T_0} > f_{T_+} \tag{5.43}$$

Combining these inequalities with (2.121) we have the following expression for the expansion differentials, κ_e and ω_e, for $\rho_{T_+} - \rho_{T_0} = \rho_e$, $\lambda_{T_+} - \lambda_{T_0} = \lambda_e$ and $\tau_{T_+} - \tau_{T_0} = \tau_e$

$$ {}_4\hbar_2 \equiv \rho_e \omega_0{}^2 \equiv -\left(\hbar \kappa_e{}^4\right)\omega_0{}^2 = -\left(\hbar \omega_e{}^2 \kappa_e{}^2\right)\kappa_0{}^2 \equiv f_e \kappa_0{}^2 . \tag{5.44} $$

$$ {}_2\hbar_2 \equiv \lambda_e \omega_0{}^2 \equiv -\left(\hbar \kappa_e{}^2\right)\omega_0{}^2 = -\left(\hbar \omega_e{}^2\right)\kappa_0{}^2 \equiv \tau_e \kappa_0{}^2 . \tag{5.45} $$

Since

$$ c \equiv \frac{\omega_0}{\kappa_0} \equiv \frac{\omega_p}{\kappa_p} \equiv \frac{\omega_e}{\kappa_e} , \tag{5.46} $$

we might surmise that this states that c is invariant, but in a special sense. In particular, it says that x_e and t_e are covariant. It does not, however, say that, for the following

$$ \omega_0\left(T_0\right) = \omega_0\left(T_+\right) \text{ or} \tag{5.47} $$

$$ \kappa_0\left(T_0\right) = \kappa_0\left(T_+\right) , \tag{5.48} $$

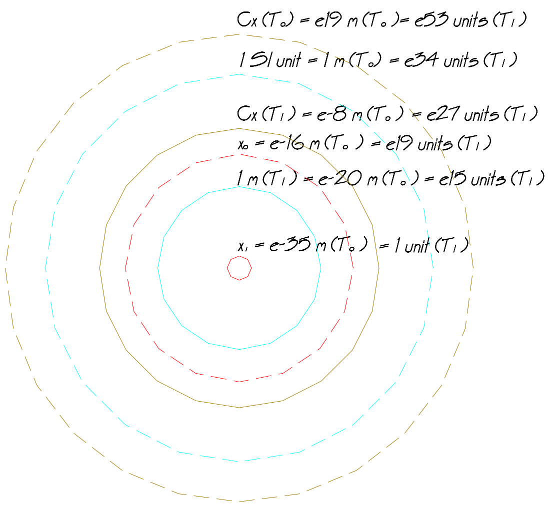
where $\omega_p = \omega_0\left(T_+\right)$ and $\kappa_p = \kappa_0\left(T_+\right)$. Therefore, the expansion rate appears to be the operation of a complex exponential function, as might be expected from the above derivations, indicating its cyclical nature. We find here the oscillation of individual quanta as the function of an overall cosmic oscillation, where Ω is the cosmic frequency, iA_c is the imaginary part of the cosmic complex amplitude and $0 < \Omega T < \frac{\pi}{2}$, given by

$$ m_0 = \frac{\hbar}{A_c e^{i(\pm \Omega T)}} = \frac{\hbar}{iA_c \,\mathbf{sin}\left(\pm \Omega T\right)} \tag{5.49} $$

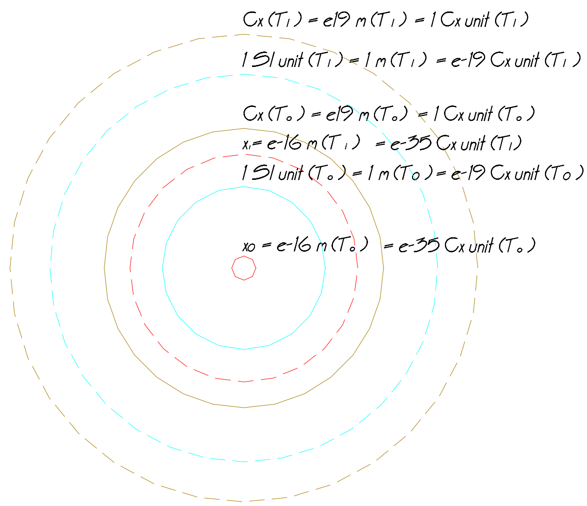
where we have used the imaginary part of Euler and in which, owing to the close approximation of ΩT to $^{\pi}/_2$ we can use x_0 in lieu of C_x in the cosine,

$$ \mathbf{cos}\,\Omega \mathrm{T} \cong \mathbf{cot}\,\Omega \mathrm{T} = \frac{x_{Planck}}{x_0} = \frac{\sqrt{T_0}}{T_0} \tag{5.50} $$

The left figure below shows the relationship in orders of magnitude of the Planck length, neutron scale, meter, and presumed cosmic extent in SI units at current cosmic time T_0. Also shown are the presumed cosmic extent and meter at the point of cosmic inception, T_I. At the right end of each parameter is the value in units of the Planck length.



Cosmic - Planck Scale



Invariance of the Fundamental Length and Cosmic Extent

The above speculations concerning the form expansion might take need not be the only interpretation of the above analysis. It appears that these parameters might be interpreted in the context of a fixed 3-space without boundary in which the apparent expansion corresponds with an isotropic contraction of the oscillators toward the Planck length, in keeping with the development of the gravitational quantum and the hyperbolic nature of the inversphere, the ultimate fate of local groups being coalescence in inertial sinks. Red shift would come from this shrinkage.

In the right figure the invariance of the relationship among C_x, the meter, and x is shown at time T_I and T_0. Note that the relationship is unchanged if I and 0 are transposed, as would be the case for cosmic expansion. In this figure, we are assuming that C_x is the fundamental unit and therefore remains unchanged, with all other values expressed in terms of it. Rotational oscillation of that extent, in conjunction with the inertia of its core, would induce the stress responsible for its quantum oscillations. x_0 has the value of the Planck length in terms of the invariant cosmic extent, and C_x and a meter, both at T_0, represent their apparent values at T_I, based on the assumption of expansion. By extention of this logic, the cosmic extent could oscillate between contraction in this manner and expansion as outlined before, with the invariance of the fundamental relationship intact throughout the oscillation.

Finally, in this regards, the condition in the right figure might be interpreted as representative of a hyperbolic 3-space without boundary of fixed extent, in which the apparent contraction or expansion is simply an artifact of negative curvature similar to an Escher print. It is not a necessary conclusion from the data that all hadronic and leptonic matter proceeds from one initial inertial font, such as a big bang. It is entirely feasible that such matter is generated by active galactic nuclei, galaxies being the largest discrete structures observed in the universe. The double rotation of the quanta is reflective of the conditions of helical stress-strain that would exist at the center of such galaxies, involving simultaneous toric annular and polar rotation, with black hole density and stress. The collimated, relativistic jets, with gamma frequency observed to issue from these loci suggest two such tori back to back at a sandwiched accretion disk, their central black holes in fact inertial fonts, generating hydrogen plasma. The lighter elements, including molecular hydrogen and helium of all isotopic configurations, with perhaps some lithium, would be expected to emerge from such high energy fonts, the heavier congregations of nuclear quanta being generally unable to withstand the relativistic pressure of generation which would necessarily exceed that of T_0.

One final word is in order with respect to special relativity. It is apparent that any instance of translational momentum increases the stress on and hence energy of an oscillation, fundamental or secondary, with a contraction of r, resulting in an increase in ω and therefore in κ, which by virtue of ת results in an increase in or relativistic augmentation of m. This development, therefore, is in keeping with the Pythagorean theorem and the Lorentz framework, though not necessarily all the conclusions drawn from the customary SR interpretations of time dilation and length contraction.

6 – Evaluations

Observed Values

For evaluation purposes, the following, which are 2002 CODATA values, or computed directly from those values, are used with the exception of those denoted by †. Asterisks indicate observed values use in computing the theoretical values of this model.

Speed of electromagnetic wave propagation*

$$c = 299,792,458\ m/s \quad * (6.1)$$

Planck's Constant of Action*

$$\hbar = 1.05457168(18)x10^{-34}\ kg\ m^2/s \quad * (6.2)$$

Newton's Gravitational Constant

$$G_N = 6.6742(10)x10^{-11}\ N \cdot m^2/kg^2 \quad (6.3)$$

Elementary Charge

$$e = 1.60217653(14)x10^{-19}\ C(kg\ m/s) \quad (6.4)$$

Fine Structure Constant

$$\alpha = 7.97352568(24)x10^{-3} \quad (6.5)$$

Neutron Mass*

$$m_n = 1.67492728(29)x10^{-27}\ kg \quad * (6.6)$$

Neutron Compton Wavelength over 2π

$$\lambda\!\!\!^{-}_{C,n} = 2.100194157(14)x10^{-16} \quad (6.7)$$

Neutron-Electron Mass Ratio, Inverse

$$\frac{m_n}{m_e} = 1838.6836598(13),\ \frac{m_e}{m_n} = 0.00054386734481(38) \quad (6.8)$$

Neutron-Proton Mass Ratio, Inverse

$$\frac{m_n}{m_p} = 1.00137841870(58), \frac{m_p}{m_n} = 0.99862347872(58) \quad (6.9)$$

Proton-Electron Mass Ratio, Inverse*

$$\frac{m_p}{m_e} = 1836.15267261(85), \frac{m_e}{m_p} = 0.00054461702173(25) \quad * (6.10)$$

Planck Scale (Area)

$$\mathrm{A}_P \equiv \frac{G_N \hbar}{c^3} = 2.611810291...x10^{-70}\ m^2 \quad (6.11)$$

Electron Angular Frequency, Inverse

$$\omega_e = \frac{c}{\lambda\!\!\!^{-}_{C,e}} = 7.7634407(52)x10^{20}\ \theta/s, \omega_e^{-1} = 1.28808867...x10^{-21} s/\theta \quad (6.12)$$

Hubble Constant†

$$H_0 = 7.2x10^4 \text{ to } 7.3(8)x10^4 m/s/mps \quad (6.13)$$

Reduced Hubble Constant†

$$H_R = 2.333x10^{-18} \text{ to } 2.368x10^{-18} \Delta m/s \quad (6.14)$$

Theoretical Values

The computed values derived from this inertial theory as follows are based on the 2002 CODATA values for c, $\hbar$, m_n and the ratio of m_n to m_e or m_p to m_e.

Inertial Constant*

$$\daleth = \frac{\hbar}{c} = 3.51767248(60)x10^{-43}\ kg\ m\ \left(kg\ m/\theta\right) \tag{6.15}$$

Fundamental Wave Number

$$\kappa_0 = \frac{m_n}{\daleth} = ƛ_{C,n}^{-1} = 4.76146453(81)x10^{15}\ \ \theta/m \tag{6.16}$$

Fundamental Angular Frequency

$$\omega_0 = c\kappa_0 = \frac{c}{ƛ_{C,n}} = 1.42745115(24)x10^{24}\ \ \theta/s \tag{6.17}$$

Fundamental Wave Transverse Momentum

$$\daleth_1 \equiv P_M = -i5.02130565(85)x10^{-19}\ kg\ m/s \tag{6.18}$$

Fundamental Wave Transverse Force

$$\daleth_2 \equiv \tau_0 = -7.167668571\mathbf{...}x10^{5}\ N \tag{6.19}$$

Fundamental Linear Inertial Density of the Spacetime Continuum

$$_2\daleth \equiv \lambda_0 = -7.975106838\mathbf{...}x10^{-12}\ kg/m \tag{6.20}$$

Fundamental Mechanical Impedance of the Spacetime Continuum

$$_1\daleth_1 \equiv Z_0 = 2.390876882\mathbf{...}x10^{-3}\ N/\left(m/s\right) \tag{6.21}$$

Fundamental Tension Stress/Volume Energy Density of the Spacetime Continuum

$$_2\daleth_2 \equiv f_0 \equiv E_1 = 1.625021173\mathbf{...}x10^{37}\ N/m^2 \tag{6.22}$$

Fundamental Quantum Spin Energy

$$^1\daleth_2 \equiv E = i1.505349565\mathbf{...}x10^{-10}\ J\left(N\cdot m\right) \tag{6.23}$$

Fundamental Quantum Power

$$^1\daleth_3 \equiv P_{\omega_0} = 2.148812979\mathbf{...}x10^{14}\ W\mathbf{(}J/s\mathbf{)} \tag{6.24}$$

Fundamental Quantum Gravitational Force

$$\frac{d\daleth_2}{d\,_2\daleth_2} \equiv G_q \equiv \frac{d\tau_0}{dT_0} \equiv \frac{1}{6\sqrt{3}\kappa_0{}^2} = 4.244309191\mathbf{...}x10^{-33}\ N \tag{6.25}$$

Newton's Gravitational Constant (Derived)

$$G_{Nq} \equiv \frac{G_q}{\lambda_0^2} = 6.673197749\mathbf{...}x10^{-11}\ N/\left(kg/m\right)^2 \tag{6.26}$$

Planck Scale (Area, Derived)

$$\frac{d_2}{d\,_2\daleth_2} \equiv \mathrm{A}_P \equiv \frac{d\mathrm{A}_0}{dT_0} \equiv -\frac{6\sqrt{3}\tau_0}{T_0^2} = -2.611848548\mathbf{...}x10^{-70}\ m^2 \tag{6.27}$$

Fundamental Charge (Raw Derivation of SI Value)

$$e_{raw} = \frac{\daleth\omega_0}{\pi} = 1.598331232\mathbf{...}x10^{-19}\ kg\ m/s \tag{6.28}$$

Natural Fundamental Charge

$$e' = \partial\omega_0^{-1} = \omega_e^{-1} = 1.288089086\mathbf{...}x10^{-21}\ \mathrm{s}/\theta \tag{6.29}$$

Fundamental Driving-Driven Frequency (Neutron-Proton Mass) Ratio, Inverse

$$\frac{\omega_0}{\omega_p} = 1.0013784732\ldots, \frac{\omega_p}{\omega_0} = 0.998623425\ldots \tag{6.30}$$

Fundamental Driven-Reflected Wave Frequency (Proton-Electron Mass) Ratio, Inverse (Based on the CODATA Neutron-Electron Mass Ratio)

$$\frac{\omega_p}{\omega_e} = 1836.152075\ldots, \frac{\omega_e}{\omega_p} = 0.000544617199\ldots \tag{6.31}$$

Fundamental Driving-Reflected Wave Frequency (Neutron-Electron Mass) Ratio, Inverse (Based on the CODATA Proton-Electron Mass Ratio)

$$\frac{\omega_0}{\omega_e} = 1838.684256\ldots, \frac{\omega_e}{\omega_0} = 0.0005438671685\ldots \tag{6.32}$$

Differential Linear Inertial Density and Cosmic Expansion Rate

$$\partial_2 \daleth \equiv \partial\lambda_0 \equiv X_e \equiv \daleth\partial\kappa_0^2 \equiv c^{-2}\daleth\partial\omega_0^2 = 2.35896879\ldots x10^{-18}\, kg/m/s, \Delta m/s \tag{6.33}$$

Differential Expansion Force

$$\partial\daleth_2 \equiv \partial\tau_0 = 0.212013542\ldots N/s \tag{6.34}$$

Hubble Rate

$$H_0 = 72{,}791.17172\ldots m/s/mps \tag{6.35}$$

Young's Modulus of the Spacetime Continuum

$$6\sqrt{3}\,{}_3\daleth_2 \equiv Y = 8.041025\ldots x10^{53}\, N/m^2 \tag{6.36}$$

Volume Potential Force Density

$$F_1 \equiv 6\sqrt{3}\,{}_4\daleth_2 \equiv \mathrm{A}_{Planck}^{-1} = 3.82871\ldots x10^{69}\, N/m^3 \tag{6.37}$$

Appendix A - Direct Product and Inverse Square Law

At least two of the principle interactions between apparently discrete bodies are governed by this law which states that the strength or magnitude of the interaction, F_Q, between two bodies is directly proportional to the product of the quantitative measure of some quality of those bodies, Q, and inversely proportional to the square of the distance of their spatial separation, d. This finds mathematical expression as

$$F_Q = \left(\frac{Q_1 Q_2}{d^2}\right) k_Q, \tag{7.1}$$

where k_Q is a constant of proportionality which may be a derivative with respect to some argument common to both side of the equation or simply a differential quantity which is integrated by the bracketed term.

If we assume that all bodies are composed of discrete portions or quanta, then the stated quality, Q_a, for each body, a, would be the number of such quanta, n_a, times the unit quality per quanta, q, or $Q_a = n_a q$. Similarly, the distance could be expressed in terms of some quantum unit r_q, so that $d = n_r r_q$. This gives

$$F_Q = \frac{n_1 n_2}{n_r{}^2}\left(\frac{q^2}{r_q^2} k_Q\right), \tag{7.2}$$

and the term in brackets can be viewed as a quantum of the interaction magnitude or force, F_Q. According to Newton's second law, force is generally defined as the mass of a body times its acceleration, positive or negative as with

$$F = ma = mv'(t) = mr''(t) \tag{7.3}$$

in which it is shown that acceleration is the change in velocity as a function of time, velocity being a change in position or state as a function of time. Mass is envisioned as a measure of the inertia or resistance to change in position of a body. In general, therefore, velocity can be viewed as the rate of change over time of any variable quality, acceleration as the rate of change in a variable rate of such changeable quality, and mass as an inverse measure of the susceptibility or a direct measure of the resistance to change of a second quality that is being changed. The third term in (7.3) indicates that force is a change in momentum over a period of time, where momentum, P_M, is such second quality, m, undergoing some change at a steady rate, v, which might possibly be zero

$$P_M = mr'(t) = mv \tag{7.4}$$

Gravitation. The magnitude of gravitational attraction is directly proportional to the product of the quality of mass, M_a, of two interacting bodies and inversely proportional to the square of the distance, d, separating their centers of mass. This finds mathematical expression in Newton's law of universal gravitational attraction generally stated as

$$F_g = \frac{M_1 M_2}{d^2} G_N. \tag{7.5}$$

where G_N is Newton's empirically determined gravitational constant.

In quantum terms this would be

$$F_g = \frac{n_1 n_2}{n_r^{\ 2}}\left(\frac{m_q^{\ 2}}{r_q^{\ 2}} G_N\right). \qquad (7.6)$$

Therefore, the number in brackets represents a quantum of gravity or

$$G_q = \left(\frac{m_q^{\ 2}}{r_q^{\ 2}} G_N\right). \qquad (7.7)$$

It is worth noting that if we use the CODATA value of the mass of the neutron for m_q, and the neutron Compton wavelength over 2π for r_q, G_q resolves to that same value of r_q divided by $6\sqrt{3}$ within a factor of 1.000014648, which is within the margin of error for G_N. This value for G_q is derived herein.

Electrostatics. The strength or magnitude of electrostatic interaction is directly proportional to the magnitude of the product of the quality of charge, q_a, of each of two interacting bodies and inversely proportional to the square of the distance separating their centers of charge. This finds mathematical expression in Coulomb's law of universal electrostatic interaction generally stated as

$$F_{es} = \frac{q_1 q_2}{d^2} k_\varepsilon = \frac{n_1 e_1 n_2 e_2}{d^2} \frac{1}{4\pi\varepsilon_0}. \qquad (7.8)$$

The right most term uses quanta of charge, e, for the direct product, where e_1 and e_2 may be of like or opposite complex sense . Thus, if the sense of both is $-i$ or $+i$, the product will be negative, whereas if the senses are opposite, the product will be positive, the positive product representing an attractive and the negative, a repulsive, force. The constant of proportionality in this case, k_ε, is equal to the inverse of 4π times the dielectric constant or permittivity of the vacuum, ε_0. Applying the same logic to the inverse square term as above, we have

$$F_{es} = \frac{n_1 n_2}{n_r^{\ 2}}\left(\frac{\pm e^2}{r_q^{\ 2}} \frac{1}{4\pi\varepsilon_0}\right), \qquad (7.9)$$

where once again the bracketed term can be seen as a quantum of force. Since force has the dimensions of $\frac{ml}{t^2}$ or mass times length over time squared, the numerator of that term might be imagined to have the dimensions of ml and the bottom of t^{-2}. In fact, if ε_0 is given the dimensions of v^{-2}, or inverse velocity squared, this could be the case.

Electrodynamics. The strength of an electromotive force between two currents of charge is directly proportional to the currents or the quantity of moving charges per second of each and inversely proportional to their distance of separation. It is not strictly speaking an inverse square law. It finds mathematical expression in the following definition of the ampere or basic unit of current.

$$\mathbf{F}_{ba} = i_b \mathbf{L} \times \mathbf{B}_a \text{, where } B_a = \frac{\mu_0 i_a}{2\pi d} \qquad (7.10)$$

For two currents running parallel this becomes

$$\frac{F_{ba}}{L} = \frac{\mu_0 i_a i_b}{2\pi d} \tag{7.11}$$

B_a is the magnetic field produced by current i_a at the location of current i_b, d is their distance of separation, μ_0 is the magnetic constant or the permeability of the vacuum and F_{ba} is the force exerted on length L of current i_b by the magnetic field B_a. That force is directed toward current i_a in the case of parallel currents, that is, those running in the same direction, and away from i_a in the case of anti-parallel currents, those running in opposite directions. It is noted that L and d are normal or perpendicular to each other. For an empirically determined value for F_{ba} of $+2 \times 10^{-7}$ Newton per meter length of current, L, at separation, d, of one meter, μ_0 is conventionally set at a magnitude of $4\pi \times 10^{-7}$ so that equal currents for both i_a and i_b, have a value of 1 Ampere. This is an arbitrary value selection for μ_0 to facilitate computation. As such, the product $\mu_0\varepsilon_0$ is equal to the inverse of the speed of light squared or c^{-2}, so that if μ_0 is given another value, ε_0 and i will adjust accordingly. What is not arbitrary is the product of the number of charge quanta, $n_{e1}\, n_{e2}$, per length and separation of currents per period of time, required to produce an F_{ba} of $+2 \times 10^{-7}$ Newton.

Since L and d are both of the same magnitude, we can speculate that for any value of $L = d$, and $i_a = i_b = 1$, F_{ba} is invariant, including at the quantum level indicated in (7.1) above of r_q. With respect to (7.11) it is apparent that $\mu_0 i_a i_b$ has the dimensions of force. We have the option, however, of assigning the current the dimensions of the square root of force and making μ_0 a dimensionless number in keeping with the comments concerning (7.9) above or of making μ_0 an inverse quantum of force and letting the current represent a count of quanta of force, n_1 and n_2, as previously done above. Therefore we have

$$F_{ba} = \frac{\mu_0 i_a i_b}{2\pi} = 2\text{ת}\alpha n_{e1} n_{e2}\,, \tag{7.12}$$

where the current has been replaced by the number of elementary charges per volt and thereby, with unit resistance per ampere, ת, (tav), is an inertial constant of dimensions mass-distance, and α is the fine structure constant. We can then assign the dimension of inverse time to each charge count. In reality, ת is a second order force differential with respect to time or frequency which is integrated by the product of the count of the charges. The final two terms of (7.12) can then be rearranged to give the more familiar

$$\frac{i_a i_b}{n_{e1} n_{e2}} \frac{1}{4\pi\alpha\varepsilon_0} = \frac{e^2}{4\pi\alpha\varepsilon_0} = \text{ת}c^2 \tag{7.13}$$

where the final term is equivalent to the quantum of action times the speed of light. It is apparent that the second term is equivalent to the bracketed term of (7.9) above with the substitution of α for the r_q^2. This gives us the option of assigning the square of some quantum unit-length dimension to α, which we will see can be done with some justification, or of making the current a count of flow per unit length. While current is generally conceived as the count of the number of charge carriers or charged “particles” operating at a given point over a unit of time, we can also think of it as the number per length of current. On a quantum level, with some rearrangement (7.13) then becomes, where α is dimensionless,

$$\frac{e^2}{r_q^2 4\pi\varepsilon_0} = \alpha\hbar c^2 \frac{1}{r_q^2} = \alpha\hbar\omega_q^2 = \alpha\hbar c^2 \kappa_q^2 \tag{7.14}$$

and we find the equivalence of (7.9).

One final observation concerning dimensions involved in mathematical operations such as taking of the product or square root, is that while the product of two linear dimensions preserves the number of dimensions in the produced area, for example, the same is not necessarily true in the case of vector operations. Thus the cross product of two forces resulting in a torque retains the number of dimensions *(mlt^{-2})* as each of the two components, though the resulting product is not quite the same quality as its components. The decomposition of such product through the square root or other factoring thus correctly results in an apparent doubling of dimensions and not necessarily, for example, the square root of force for each component. Thus charge and current can be modeled as "forces" which operate tangentially to produce a radial "force" effect. The quotation marks are used to denote dimensional, though not necessarily technical, equivalence.

Appendix B – Wave Transmission at a Discontinuity

We might look to a function that concerns the reflection and transmission of waves at a discontinuity of the propagating medium, to better understand the boundary conditions governing charge generation. Such discontinuity might be a change in inertial density of the medium itself or a change in tension or perhaps shearing stress along its span. We can assume initially that the transverse displacement of the medium on both sides of the discontinuity is the same, so that the frequency of a wave crossing the boundary is unchanged. The following is taken from Elmore and Heald, Physics of Waves.

For a one dimensional model of wave transmission at a discontinuity, the boundary conditions require that in addition to the transmission of the initial wave with altered amplitude, a reflected wave must be generated. As a result of these conditions we have the following, in which A_1 is the amplitude of the incident wave, $\breve{A}_2$ is the complex amplitude of the transmitted wave, and κ_2, its wave number, $\breve{B}_2$ is the complex amplitude of the reflected wave, and κ_1 is the wave number of the initial and reflected waves.

$$A_1 + \breve{B}_1 = \breve{A}_2 \tag{7.15}$$

$$\kappa_1 A_1 + \kappa_1 \breve{B}_1 = \kappa_2 \breve{A}_2 \tag{7.16}$$

From this we can solve for the amplitude reflection, $\breve{R}_a$, and transmission, $\breve{T}_a$, coefficients or ratios, and for the power reflection, R_p, and transmission, T_p, coefficients, which depend only on the properties of the medium. We can see from (2.91) that if the frequency is constant, the impedance Z relationships will be identical to that of the wave numbers.

$$\breve{R}_a \equiv \frac{\breve{B}_1}{A_1} = \frac{\kappa_1 - \kappa_2}{\kappa_1 + \kappa_2} = \frac{Z_1 - Z_2}{Z_1 + Z_2} \tag{7.17}$$

$$\breve{T}_a \equiv \frac{\breve{A}_2}{A_1} = \frac{2\kappa_1}{\kappa_1 + \kappa_2} = \frac{2Z_1}{Z_1 + Z_2} \tag{7.18}$$

$$R_p \equiv \frac{\frac{1}{2}\lambda_1 c_1 \omega^2 B_1^2}{\frac{1}{2}\lambda_1 c_1 \omega^2 A_1^2} = \frac{B_1^2}{A_1^2} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2 \tag{7.19}$$

$$T_p \equiv \frac{\frac{1}{2}\lambda_2 c_2 \omega^2 A_2^2}{\frac{1}{2}\lambda_1 c_1 \omega^2 A_1^2} = \frac{\lambda_2 c_2 A_2^2}{\lambda_1 c_1 A_1^2} = \frac{4 Z_1 Z_2}{\left(Z_1 + Z_2\right)^2} \tag{7.20}$$

It should further be stated that the requirements of conservation of energy and power dictate that

$$R_P + T_P = 1 \tag{7.21}$$

Appendix C – Vector Orthogonality

The 4-vector equation can be generalized to n dimensions for an ortho-normal space as,

$$\sqrt{n-1}\,x_{0i} = \sqrt{\underline{x}_n^2 - x_{0nth}^2} \tag{7.22}$$

Appendix D - Exponentiation

Calculus is the study of the rate of change in one variable quantity, conventionally denoted by a *y*, which is held to be a function, *f*, wholly or partially, of another variable, generally denoted by an *x* or sometimes a *t*. This underlying functional relationship between the variables is denoted by

$$y = f(x) \text{ or } y = f(t). \tag{7.23}$$

In the case of a partial function, a function of more than one variable, we write

$$y = f(x,t). \tag{7.24}$$

Thus, with (7.23) when $x = a$, $y = b$, and with (7.24) when $x = a$ and $t = b$, $y = c$. *a*, *b* and *c* are arbitrary symbols standing for unknown quantities of the stated variable *x*, *t* and *y*, and depending on the context and circumstance *a*, *b* and *c* may in fact be the same or of equal value.

The underlying functional relationship or function does not necessarily indicate that *x* causes *y* or that *y* is the <u>operational</u> function of *x*. While this may be so in the case of some physical and organizational conditions, in general terms the function simply indicates that when *x* has the value of *a*, *y* is always, within the context determined by *f*, uniquely observed to have the value of *b*.

Thus, given a right triangle of variable angle, α, but fixed, unit length hypotenuse, the cosine can be stated as a function of the length of the adjacent side, *a*, and the length of *a* can be stated as an inverse function of the cosine. In the language of mathematics, we would say that the cosine function maps the value of *a* onto the cosine and the inverse function maps the value of the cosine onto *a*. This concept of mapping reflects the fact that any function that we might consider can be visualized and charted against the backdrop of an orthogonal co-ordinate system.

Thus it may equally be true that

$$x = f(y) \text{ or } t = f(y). \tag{7.25}$$

Note that it is not generally stated that

$$x,t = f(y), \tag{7.26}$$

although that may in fact be the case. If *x* is the adjacent side of α and *t* is the opposite, they both vary with respect to some variation in the angle, $y = \alpha$.

While it may be of interest to know the value of *y* for any value of *x* or *t*, it is often of equal or greater interest to know the rate at which *y* is changing for any value of *x* or *t*. This rate of change or ratio of variability of one quantity with respect to another is known as the derivative function, f', of *y* with respect to *x* or *t*, or

$$\frac{dy}{dx} = f'(x) \text{ or } \frac{dy}{dt} = f'(t). \tag{7.27}$$

The quantity *dy* is the differential amount of change in *y* that occurs for every differential amount of change, *dx*, in *x*. While *dy* and *dx* are customarily envisioned as being infinitesimally small, they are generally not small by the same proportions, and are indeed expressed as the change in *y* in units of that quality for every change of <u>one unit</u> of *x*. Hence they are often used in partial derivative form as direction cosines, where by implication, ∂x would be the hypotenuse of a right triangle of unit value and ∂y is the adjacent side. Here f' constitutes another function of *x*, with the prime notation indicating that it is a derivative function of *f(x).* The single prime is the rate of change in *f(x)* commensurate with a change in *x*. If it is a function with respect to time, *f(t),* i.e. a rate of change over some unit of time, it is a speed, or if a direction is specified, a velocity.

If the rate of change in *f(x)* or *f(t)* is not steady or constant, then we have a second derivative of these functions, denoted by a double prime

$$\frac{d^2y}{dx^2} = f''(x) \text{ or } \frac{d^2y}{dt^2} = f''(t). \tag{7.28}$$

The change in velocity, acceleration, which is the second derivative with respect to time, $f''(t)$, is commonly encountered and understood. The equivalent with respect to *x,* also a type of acceleration, is a change in the intensity or magnitude of some derivative, $f'(x)$, with each change in *x*. Thus if $f'(x)$ represents the slope of a mountainside, the change in elevation per change in horizontal displacement, when the slope is a constant pitch, then $f''(x)=0$, that is, it does not exist. If the slope gets steeper as the mountain is climbed, then the second derivative is positive. In physics this second derivative of *x* is called the Laplacian. A force embodying the inverse square law is an instance of the second derivative.

If the acceleration, the change in $f'(x)$ or $f'(t)$, is not constant, then we have a third derivative of these functions, denoted by the triple prime or

$$\frac{d^3y}{dx^3} = f'''(x) \text{ or } \frac{d^3y}{dt^3} = f'''(t). \tag{7.29}$$

With respect to $f''(t)$, this acceleration of acceleration is known as jerk. Any change from a position of rest involves an element of jerk, since the acceleration from zero to any finite velocity is not instantaneous or constant. With respect to a mountainside, if the slope increases exponentially with the climb, instead of at a steady rate of say 50 meters per kilometer of horizontal distance covered, then the third derivative is functioning.

In (7.28) and (7.29) it will be noticed that the differential with respect to *y* is preceded by the order of the derivative, while the differential with respect to *x* and *t* is followed by the order or exponent of the derivative. This is due to the fact that the latter variables are actual squares and cubes, that is powers of the differentials, while the order of the dependent differential of *y* indicates the change in *y* attributed to the independent variable of the same order. The dimensionality of *y* is always of what ever happens to be the inherent dimensionality of the quality *y* represents. If *y* is a force, $d^n y$ will itself have units of force. Notice that $f^n(t)$, then would have units of force per time to the nth power.

A geometric example will perhaps make this clear. While the following may not be the customary context for the second and higher order derivatives, it is a legitimate instance of such. The equation for the volume of a cube, in which the volume, *V*, is a function of the length of one side, all sides being by definition equal, is

$$V = f(x) = x^3 . \tag{7.30}$$

The inverse function is

$$x = f(V) = V^{\frac{1}{3}} . \tag{7.31}$$

A change in volume, *dV*, is still a three dimensional quality, and we might be tempted to say that it is equal to dx^3, which in a certain context it is. If we want to express that change as a derivative function, however, we must use the definition of a derivative, which gives the original function plus the differential change as

$$V + dV = (x + dx)^3 = x^3 + 3x^2dx + 3xdx^2 + dx^3 . \tag{7.32}$$

Subtracting (7.30), the terms in square brackets, gives the differential

$$V + dV - [V] = x^3 + 3x^2dx + 3xdx^2 + dx^3 - \left[x^3\right] \tag{7.33}$$

$$dV = 3x^2dx + 3xdx^2 + dx^3 \tag{7.34}$$

This last equation is a bit opaque, however, as it assumes that one of the corners of the cube is at the origin of a co-ordinate system and injects a corresponding bias into the derivation, which may or may not be warranted. If we locate the origin at the center of cube, while still assuming each side aligned with the co-ordinates, we must assign the differential change in *x* to each end of a length and we have instead of (7.32)

$$\begin{aligned} V + dV - [V] &= (x + 2dx)^3 - \left[x^3\right] \\ &= x^3 + 6x^2dx + 12xdx^2 + 8dx^3 - \left[x^3\right] \end{aligned} \tag{7.35}$$

$$dV = 6x^2dx + 12xdx^2 + 8dx^3 \tag{7.36}$$

From this last derivation, it is immediately clear that the differential volume is made up of the six sides of the cube, the twelve edges, and the eight vertices or corners, all times an infinitesimal length of varying orders. The first order derivative, corresponding to the six square sides of the cube of measure x^2 and the one usually deemed to have the greatest significance, is explicitly stated as

$$f'(x) = \frac{d^1V}{dx^1} = 6x^2 . \tag{7.37}$$

The second order derivative, corresponding with the twelve edges of the cube, consisting of twelve line segments of length *x*, is

$$f''(x) = \frac{d^2V}{dx^2} = 12x^1 . \tag{7.38}$$

The third order derivative, corresponding to the eight vertices, each of zero, or technically, vanishing dimension, is

$$f'''(x) = \frac{d^3V}{dx^3} = 8x^0 . \tag{7.39}$$

Thus for the total derivative or (increasing) change in *V* with respect to a change in *x*, we have

$$\sum_{n=1}^{3}\frac{d^nV}{dx^n}=\frac{d^1V}{dx^1}+\frac{d^2V}{dx^2}+\frac{d^3V}{dx^3}=6x^2+12x+8\,. \qquad (7.40)$$

If *V* were decreasing, and *dx* were a decrement, the change would be

$$-\sum_{n=1}^{3}\frac{d^nV}{dx^n}=-\frac{d^1V}{dx^1}+\frac{d^2V}{dx^2}-\frac{d^3V}{dx^3}=-6x^2+12x-8\,. \qquad (7.41)$$

Note that the decrease from the initial condition is indicated by the negative sense of the summation, but that the magnitude of the sum in this case is of a positive 6 squares, minus 12 line segments, while adding back the 8 vertices. In more elucidating fashion the magnitude of the derivative becomes

$$\sum_{n=1}^{3}\frac{d^nV}{dx^n}=\frac{d^1V}{dx^1}-\left(\frac{d^2V}{dx^2}-\frac{d^3V}{dx^3}\right)=6x^2-\left(12x-8\right). \qquad (7.42)$$

While it is clear that the contribution of the 8 vertices is not effected by the value of *x*, it is obvious that as *x* increases, the contribution to the sum made by the edges increases linearly with *x*, while the contribution made by the surface squares increases exponentially, specifically by the power of 2.

If *dx* is not quite zero, but exceedingly small compared to *x*, then it is apparent that the order of the derivatives is a fair appraisal of each component's contribution to *dV*. The additional volume is predominantly surface differential. In fact, at the limit as *dx* approaches 0, each order is exponentially greater that the next order in succession. However, if the components are allowed to increase exponentially beyond the value of *x*, the situation inverts itself.

If we think of the original cube, still positioned about the origin, as having some very small unit edges of length *x*, where *x* is the smallest imaginable length, and make the change, *dx*, exceedingly great, in fact approaching infinity, then the first order of six squares constitutes the six sense-axes of a 3-D space, the second order, the twelve edges, constitutes the twelve quadrants of the x-y, y-z, and z-x planes, while the third order of the eight vertices become the 3-D octants of the co-ordinate system.

In the above cubic scenario, it is apparent that the numerical coefficients, which in a standard development of the calculus arise through the operation of the binomial expansion as with (7.32), are actually inherent aspects of the specific cubic geometry. The derivation consists of a division of each of the orders of differentiation by *x, n* times, where *n* indicates the order of each term. As such it represents a reduction in the power of each term by 1 for each time or order. Alternatively, we can view this as a multiplication for each instance of differentiation of $x^{-1}dx$. Thus, with the observation that a change in the volume of a cube must occur at its 3 boundary elements, i.e. faces, edges and vertices, (7.36) can be arrived at by

$$\begin{aligned} dV &= \sum_{n=1}^{3} \frac{d^{n}V}{dx^{n}} dx^{n} = \sum_{n=1}^{3} \frac{{}^{n}V}{x^{n}} dx^{n} = \sum_{n=1}^{3} {}^{n}V \left(\frac{dx}{x} \right)^{n} \\ &= 6x^{3} \left(\frac{1}{x} dx \right)^{1} + 12x^{3} \left(\frac{1}{x} dx \right)^{2} + 8x^{3} \left(\frac{1}{x} dx \right)^{3} \\ &= 6x^{2} dx + 12x dx^{2} + 8dx^{3} \end{aligned} \tag{7.43}$$

Notice that this describes an arrangement of 27 cubes, 3 x 3 x 3, with the original cube of volume V at the center, and that the total number of elements in the surface is 26, corresponding to the 26 adjacent cubes. The process of differentiation reduces each of these cubes exponentially, according to its relationship to the center cube. The n in ${}^{n}V$ indicates both the magnitude and the geometry of the coefficients that arise through the polynomial expansion. As there are two boundaries to any interval x, and an interval of equal magnitude to x at each boundary, x_b, a binomial of power n is

$$\left(x + 2x_b \right)^{n} = 1x^{n} + \left(3^{n} - 1 \right) x^{n-m} x_b^{m} . \tag{7.44}$$

As an example, a 4-D hypercube expansion is

$$\left(x + 2x_b \right)^{4} = 1x^{4} + 80x^{n-m} x_b^{m} = 1x^{4} + 8x^{3} x_b^{1} + 24x^{2} x_b^{2} + 32x^{1} x_b^{3} + 16x^{0} x_b^{4} . \tag{7.45}$$

Making the substitution, $\underset{\mathbf{lim}\to 0}{x_b} = dx$ for the case of a 4-D equivalent to (7.43) gives

$$\begin{aligned} \left(x + 2dx \right)^{4} - x^{4} &= 80x^{n-m} dx^{m} = 80x^{n} \left(\frac{dx}{x} \right)^{m} \\ &= 8x^{4} \left(\frac{dx}{x} \right)^{1} + 24x^{4} \left(\frac{dx}{x} \right)^{2} + 32x^{4} \left(\frac{dx}{x} \right)^{3} + 16x^{4} \left(\frac{dx}{x} \right)^{4} \\ &= 8x^{3} dx^{1} + 24x^{2} dx^{2} + 32x^{1} dx^{3} + 16x^{0} dx^{4} \end{aligned} \tag{7.46}$$

As this is obviously a logarithmic operation, as indicated by the bracketed terms, where

$$x^{n} = y \;\; \therefore \;\; \mathbf{log}_x \, y = n \tag{7.47}$$

and as the derivative of the natural log is

$$d \, \mathbf{ln} \, x = d \, \mathbf{log}_e \, x = \frac{1}{x} dx , \tag{7.48}$$

we can recast (7.43) as

$$\begin{aligned} dV &= \sum_{n=1}^{3} \frac{d^{n}V}{dx^{n}} dx^{n} = \sum_{n=1}^{3} {}^{n}V \left(d \, \mathbf{ln} \, x \right)^{n} \\ &= 6x^{3} \left(d \, \mathbf{ln} \, x \right)^{1} + 12x^{3} \left(d \, \mathbf{ln} \, x \right)^{2} + 8x^{3} \left(d \, \mathbf{ln} \, x \right)^{3} \end{aligned} \tag{7.49}$$

This last observation suggests a fundamental tie-in between the derivative of a polynomial function and that of the natural logarithm.

With this in mind, it is apparent that the matter of differentiation is closely related to the subject of exponentiation, and appears to consist of reduction by one power or order of exponentiation for each order of differentiation. In the case of the cube, it is clear that each order indicates an orthogonal reduction, from cube to square, to line segment to point.

We might wonder if this is the case for all functions of the form of (7.23). We might, for example, have a function that relates the perimeter of a square to its edge. The resulting equation and its derivative are

$$\begin{aligned} &P = 4x \\ &P + dP - [P] = 4(x + dx) - [4x] \\ &dP = 4dx \\ &\frac{dP}{dx} = 4 \end{aligned} \tag{7.50}$$

What is different in this case, is that there is no apparent second derivative. In other words, for

$$\frac{dP}{dx} = f'(x) = 4 \tag{7.51}$$

the function $f'(x)$ is not affected by the value of *x*. It is a constant and does not involve a rate of change of $f'(x)$ with respect to a change in *x*.

Any function that does not change has a first order derivative of 0. This does not mean, however, that it might not have a second order derivative. Metaphorically speaking, the water swirling around a drain might be described by some function that maps its motion around the plane of the water's surface. At the point at which it becomes vertical and disappears down the drain, effectively leaving the dimensional space of the tub, the derivative with respect to change in that space vanishes. Obviously there still must be some function describing its motion vertically and perhaps even horizontally once it has entered the plumbing system, albeit, in terms of that other dimension. The key is to realize that (7.51) actually should be written

$$\frac{dP}{dx} = f'(x) = 4x^0 = 4(1) = 4\,\mathbf{ln}_x\, x \tag{7.52}$$

$$\begin{aligned} &\therefore \frac{d^2P}{dxd\,\mathbf{ln}_x\, x} = 4 \\ &\frac{d^2P}{dx^2} = f''(x) = \frac{4}{x} \end{aligned} \tag{7.53}$$

There is another condition, however, in which, although there is a changing rate of change, there is no <u>apparent</u> change in the rate of change, i.e. no <u>apparent</u> second derivative, and that is the case of the exponential function, specifically of the natural base e, inversely related to the natural logarithmic function. The exponential function is its own derivative, of whatever order we might envision, where

$$f(x) = y = e^x \text{ iff } \mathbf{ln}\, y = x \tag{7.54}$$

so that

$$f'(x) = D_x\left[f(x)\right] = D_x e^x = e^x \tag{7.55}$$

where D_x is a differential operator, that is, it operates on f to produce f'. Thus since

$$D_x'' e^x = D_x\left[D_x e^x\right] = D_x\left[e^x\right] = e^x \tag{7.56}$$

the difference between any two orders of differentiation, in fact, between any order and the exponential function itself is

$$D_x'' e^x - D_x e^x = D_x e^x - e^x = e^x - e^x = 0\,. \tag{7.57}$$

and

$$\frac{e^x}{D_x^n e^x} = 1 \tag{7.58}$$

We might wonder what significance this has, since subtracting one order derivative from another is rather like subtracting oranges from apples. They are two different types of entity, just as a point is a different type of entity than a line segment or length, which is itself different from an area, itself different from a volume. In fact, there are generally held to be an infinite number of points in a length, lengths in an area, and areas in a volume, i.e. of dimensions in the next higher order of dimension, so the subtraction of the lesser from the greater leaves the latter substantially unchanged.

This then is the point in (7.58). The dimensional identity of each order of differentiation is the same as that of the basic function, $f(x) = y$, since it is the exponent itself that is variable and not the base. If we compare this condition with that of the cube and its derivative orders, in which the relative contribution of each order to the overall change in V is dependent on the ratio of $dx^n{:}dx$, we see that for an exponential change, the relative contribution of each order at the limit is unaffected by the change in x, or as it is often the variable used in this context, t. The ratio in the exponential case is always 1, hence the apparent lack of change.

A doubling of the sum of the lengths of the edges and a quadrupling of each surface area results in an eightfold increase in the volume of the cube. Note that there is no change in the number of boundary elements and their angular configuration, which is the defining condition of the cube. Using a combinatorial or additive approach to creating a change in the cube, then, we see that it is more economical to augment the edges to move the vertices further apart, which defines the volume change, than to fill the cube with volume, since a unit of length is orders of magnitude less than a unit of volume. Yet there is no conformal or topological difference between a cube of unit volume and one of volume 8, something we intuitively understand. The matter of scale only attains significance within a combinatorial or economical approach.

Hence in a continuum analysis, where the elements of various dimensions are integrally related, i.e. non-combinatorially, if we had a cube experiencing a continuous exponential change, each of the elements in its boundary, the faces, edges and vertices, would increase proportionally to its order with the change in volume, each derivative order increasing in proportion as the exponent of x or t. In such event, using the value of $x = \sqrt[3]{V}$ as our standard, all orders show the same exponential change and the whole is relatively, or perhaps better stated, intrinsically unchanged. It is only within the context of some extrinsically determined property, such as some external standard of length or density, i.e. volume, surface or linear, that change is registered or observed.

This is intuitively understood, especially in the preparation of scaled engineering drawings and models and other graphic representations. It is also known rarely to occur in the physical world in which physical forms result from the combination of discrete units or building blocks of matter. Adult humans do not generally look like babies three to four times their original length. Equally true, most tree girth-to-height ratios increase with growth. On the other hand, most celestial bodies of any size assume a generally spherical shape, irrespective of their volume. Rephrasing (7.43)

$$\begin{aligned} dV &= \sum_{n=1}^{3} \frac{d^nV}{dV^{\frac{n}{3}}} dV^{\frac{n}{3}} = \sum_{n=1}^{3} \frac{{}^nV}{V^{\frac{n}{3}}} dV^{\frac{n}{3}} = \sum_{n=1}^{3} {}^nV \left(\frac{dV^{\frac{1}{3}}}{V^{\frac{1}{3}}} \right)^n \\ &= 6V\left(\frac{dV^{\frac{1}{3}}}{V^{\frac{1}{3}}} \right)^1 + 12V\left(\frac{dV^{\frac{1}{3}}}{V^{\frac{1}{3}}} \right)^2 + 8V\left(\frac{dV^{\frac{1}{3}}}{V^{\frac{1}{3}}} \right)^3 \\ &= 6\left(V^{\frac{3}{3}-\frac{1}{3}} \right) dV^{\frac{1}{3}} + 12\left(V^{\frac{3}{3}-\frac{2}{3}} \right) dV^{\frac{2}{3}} + 8\left(V^{\frac{3}{3}-\frac{3}{3}} \right) dV^{\frac{3}{3}} \\ &= 6V^{\frac{2}{3}} dV^{\frac{1}{3}} + 12V^{\frac{1}{3}} dV^{\frac{2}{3}} + 8V^{\frac{0}{3}} dV^{\frac{3}{3}} \end{aligned} \tag{7.59}$$

From another but still exponential perspective, in terms of our initially outlined derivative orders, this indicates that the magnitude of displacement, velocity, acceleration, and jerk might be equal or

$$f'''(x) - f''(x) = f''(x) - f'(x) = f'(x) - f(x) = 0\,. \tag{7.60}$$

This is essentially the same equation as (7.57) and indicates that a relationship of this type is exponential in nature.

We do not have to look far for another familiar instance of such. While change is generally equated with motion and thereby with displacement or translational change of position, rotation presents an instance of motion without translational displacement, taking the position of the rotating body as a whole. It is in a sense change without change, and is elegantly presented using the Euler identity, which involves the exponential expansion of an imaginary logarithm or

$$e^{ix} = \mathbf{cos}\, x + i\,\mathbf{sin}\, x = y \tag{7.61}$$

As with the exponential function of (7.54), the domain of *x* is the real number line, but in this case, instead of the range of $y > 0$, *y* oscillates over the range of $-1 \le y \le 1$ if we consider only the real component, for each change in *x* of 2π, our angles and the “natural” unit of *x* in this case being in radians. Otherwise *y* must be a complex number whose range is a circle centered on the *x*-*y* origin in the complex plane, in this case of implied radius $r = 1$. If we then map *y* to the real *x*-*y* plane, by multiplying *y* times its complex conjugate, where *y* then gives the radius *r* and *x* is the count of the rotations, the range of *y* will be a horizontal line crossing the *y* axis at $y = r = 1$. Note that a circle of fixed radius from the origin maps as a horizontal line segment of 2π length. Since the derivative of such a line is zero, the rate of change of the rotation is zero, at least in this mapping. The only variable in such case might be the velocity of the rotation which might be reflected in the scale, the density, of the real number line. Presumably a denser placement of the integers would represent a greater velocity.

The use of a radian as the unit measure of *x* makes the equation self normalizing, that is, it sets the *x* and *y* axes of a co-ordinate system against which we might plot the function to the same scale. Assuming a rotational amplitude or modulus, i.e. the radius, *r*, equal to the hypotenuse, selection of a unit value for the *y* axis for a cosine of 1 automatically dictates the unit length for the *x* axis, since a radius, *r*, and a rotational arc of one radian measured at a distance *r* from the center are of equal length. We can then envision *x* as the distance traveled by a point *P* on the circumference of a rotating disk or equator of a sphere of radius *r*, but it might simply be a point in space that is revolving about some center of oscillation which is also the polar origin.

Thus for any value of *x*,

$$\frac{x}{2\pi} = n + \frac{\varphi}{2\pi} \tag{7.62}$$

where *n* is an integer number of rotational cycles, $-\infty < n < +\infty$, and where φ is a remainder angle or phase in which $0 < \varphi < 2\pi$. As we shall soon see, we might also state

$$\frac{x}{\frac{\pi}{2}} = n + \frac{2\varphi}{\pi} \tag{7.63}$$

where *n* is the count of the number of times that $|y|$ equals one.

If we want to express *x* in some conventional unit such as meters, we simply multiply it by the number of meters per radian and *x* will be in units of meters. In such case, in order to convert *x* back to normalized units, (7.61) becomes

$$e^{i\kappa x} = \mathbf{cos}\,\theta + i\,\mathbf{sin}\,\theta = y \tag{7.64}$$

where κ is the angular wave number or number of arc radians per unit of length and

$$\kappa = \frac{\theta}{x}. \tag{7.65}$$

A similar approach for *t* gives us

$$e^{it} = \mathbf{cos}\,t + i\,\mathbf{sin}\,t = y \tag{7.66}$$

where time is in natural units or radians, and for conversion from conventional units of time,

$$e^{i\omega t} = \mathbf{cos}\,\theta + i\,\mathbf{sin}\,\theta = y \tag{7.67}$$

where ω is the angular frequency or number of arc radians per unit of time and

$$\omega = \frac{\theta}{t}. \tag{7.68}$$

Such a condition would apply to a standing wave of fixed angular frequency.

For a wave of fixed frequency traveling from a propagating source, we can combine the two to get

$$e^{i\theta} = e^{i(\kappa x + \omega t)} = \mathbf{cos}\,\theta + i\,\mathbf{sin}\,\theta = y \tag{7.69}$$

where it is understood that x and t are of ambivalent sense.

Finally by extending the exponent of e to complex numbers we have,

$$e^{r \pm i\theta} = e^r e^{\pm i\theta} = e^r e^{\pm i(\kappa x + \omega t)} = R\left(\mathbf{cos}\,\theta \pm i\,\mathbf{sin}\,\theta\right) = y \tag{7.70}$$

$$e^{-r \pm i\theta} = e^{-r} e^{\pm i\theta} = e^{-r} e^{\pm i(\kappa x + \omega t)} = \frac{1}{R}\left(\mathbf{cos}\,\theta \pm i\,\mathbf{sin}\,\theta\right) = y, \tag{7.71}$$

and we can see that (7.69) is simply a special case of these last two in which $R = 1$. Assuming that $R > 1$, using $y = r = \sqrt{a^2 - (ib)^2}$, (7.70) now maps to the x-y real plane as a horizontal line greater than $y = 1$, and (7.71) maps to a line between the x axis and the line $y = 1$. The argument, as the middle term of (7.64) is called, conceals the fact that the sense of the angle θ determines whether $i\mathbf{sin}\theta$ is positive, changing in a counterclockwise sense, or negative, in a clockwise sense. Thus we could apply a convention in which the negative sense of $i\mathbf{sin}\theta$ maps (7.70) and (7.71) to horizontal lines crossing the negative y axis. The rotation velocity and frequency would then switch sense.

Let us examine the function

$$y = f(W) = We^W. \tag{7.72}$$

Inverting the function so that

$$W = f(y) \tag{7.73}$$

gives

$$f(y)e^{f(y)} = y. \tag{7.74}$$

We can find that

$$f(y)e^{f(y)} = W(n)e^{W(n)} = y \tag{7.75}$$

where $W(n)$ is related to the Lambert W function and in fact is identical to that function for the principal branch, integer values $n > 0$, or Lambert W(0, n>0). As will be shown, while the Lambert W function is complex for all values of $n < 0$, for all values of $-\infty \le n \le +\infty$ $W(n)$ is real and $W(n) = -W(-n)$. Further, we define

$$W(n) = n\,\mathbf{ln}\,x. \tag{7.76}$$

Therefore, (7.75) becomes

$$n\,\mathbf{ln}\,xe^{n\,\mathbf{ln}\,x} = n\,\mathbf{ln}\,x\left(x^n\right) = U_n n = y. \tag{7.77}$$

where U_n is the co-efficient of n needed to produce x for any value of y.

Inverting the function to its original

$$y = nx^n \mathbf{ln}\, x \tag{7.78}$$

and differentiating gives

$$\begin{aligned} dy &= \left(nx^n\right) d\, \mathbf{ln}\, x + \left(n\, \mathbf{ln}\, x\right) nx^{n-1} dx \\ &= \left(nx^n\right) \frac{dx}{x} + \left(n\, \mathbf{ln}\, x\right) nx^{n-1} dx \end{aligned} \tag{7.79}$$

with the derivative

$$\frac{dy}{dx} = \left(1 + n\, \mathbf{ln}\, x\right) nx^{n-1} \tag{7.80}$$

If we substitute for the natural log derivative, we have instead

$$\frac{dy}{d\, \mathbf{ln}\, x} = \left(1 + n\, \mathbf{ln}\, x\right) nx^n \,. \tag{7.81}$$

From (7.76) it follows that

$$\frac{dy}{dx} = \left(1 + W\left(n\right)\right) nx^{n-1} \tag{7.82}$$

and with a little foresight, we might imagine that this hides a complex function, as with

$$\begin{aligned} \frac{dy}{dx} &= \left(1 + iW\left(n\right)\right) nx^{n-1} \\ \frac{dy}{d\, \mathbf{ln}\, x} &= \left(1 + iW\left(n\right)\right) nx^n \end{aligned} \tag{7.83}$$

Returning to (7.76) for that case in which $U_n = 1$, $y = n$ and the normalized value of $W(n)$ is

$$W_0\left(n\right) = n\, \mathbf{ln}\, x_n \,, \tag{7.84}$$

where the n in the subscript of x relates that value of x as the unique normalizing value for $W_0\left(n\right)$ and we have

$$W_0\left(n\right)\left(x_n^n\right) = n\, \mathbf{ln}\, x_n \left(x_n^n\right) = n = \frac{y}{U_n} \,. \tag{7.85}$$

It follows that

$$\frac{W_0\left(n\right)}{n} = \mathbf{ln}\, x_n = \frac{1}{x_n{}^n} = \kappa_n{}^n = \frac{y}{nU_n x_n{}^n} \,. \tag{7.86}$$

As before, substituting t for x, the equivalent for (7.86) is

$$\frac{W_0\left(n\right)}{n} = \mathbf{ln}\, t_n = \frac{1}{t_n{}^n} = \omega_n{}^n = \frac{y}{nU_n t_n{}^n} \tag{7.87}$$

Finally, continuing in that vein

$$\omega_n = \frac{1}{t_n} = \left(\frac{W_0(n)}{n}\right)^{\frac{1}{n}} = \frac{1}{x_n} = \kappa_n \qquad (7.88)$$
$$= (\mathbf{ln}\, t_n)^{\frac{1}{n}} = (\mathbf{ln}\, x_n)^{\frac{1}{n}}$$

Since for that case in which $n = 0$, (7.86) becomes

$$\mathbf{ln}\, x_0 = \frac{W_0(0)}{0} = \frac{1}{x_0^{\ 0}} = \kappa_0^{\ 0} = 1, \qquad (7.89)$$

apparently

$$x_0 = e = e_0 \text{ and } \kappa_0 = e^{-1} = e_0^{-1} \qquad (7.90)$$

and the dividend in the first term of (7.86) must be a continuous function which approaches 0 as n approaches 0. Thus for any n, we have a fundamental base e_n in which it can be stated

$$\mathbf{ln}\, x_n = \mathbf{ln}_0\, e_n = \frac{W_0(n)}{n} = e_n^{\ -n} = e_{-n}^{\ \ n} = e_{\kappa n}^{\ \ n}. \qquad (7.91)$$

and for the inverse of x_n

$$\mathbf{ln}\, x_n^{-1} = \mathbf{ln}_0\, e_n^{-1} = \mathbf{ln}_0\, e_{-n} = \frac{W_0(-n)}{n} = -e_n^{\ -n} = -e_{-n}^{\ \ n} = -e_{\kappa n}^{\ \ n}. \qquad (7.92)$$

where we define, for conceptual reasons,

$$e_n^{\ -n} \equiv e_{-n}^{\ \ n} \equiv e_{\kappa n}^{\ \ n}. \qquad (7.93)$$

It is noted that the natural log in the second term is specified to apply to the case of (7.89) so that for any value x, for the conventional natural log x, or **ln** x,

$$\mathbf{ln}\, x = \underset{n=0}{\mathbf{ln}_n}\, x = \mathbf{ln}_0\, x. \qquad (7.94)$$

Thus for any e_n, we have

$$\mathbf{ln}_n\, e_n = e_n^{\ n}\, \mathbf{ln}_0\, e_n = 1 \qquad (7.95)$$

and

$$\mathbf{ln}_n\, x = e_n^{\ n}\, \mathbf{ln}_0\, x = y = U_n n. \qquad (7.96)$$

In continuation, we have

$$e_n^y = e_0^{\left(\frac{y}{e_n^n}\right)} = e_0^{\ y e_{-n}^{\ n}} = x, \qquad (7.97)$$
$$e_0^{\ y} = e_n^{\ y e_n^n} = e_{-n}^{\ -y e_n^n} = e_n^{\ y e_{-n}^{\ -n}} = e_{-n}^{\ -y e_{-n}^{\ -n}}, \qquad (7.98)$$
$$e_0^{\ -y} = e_{-n}^{\ y e_n^n} = e_{-n}^{\ y e_{-n}^{\ -n}} = e_n^{\ -y e_n^n} = e_n^{\ -y e_{-n}^{\ -n}} \qquad (7.99)$$

and

$$\mathbf{ln}_0\, x = \frac{\mathbf{ln}_n\, x}{e_n^{\ n}} = e_{-n}^{\ \ n}\, \mathbf{ln}_n\, x = e_{-n}^{\ \ n} y = e_{-n}^{\ \ n} U_n n, \qquad (7.100)$$

It follows with respect to derivatives that

$$\frac{1}{x} = \frac{d\,\mathbf{ln}_0\, x}{dx} = e_{-n}{}^{n} \frac{d\,\mathbf{ln}_n\, x}{dx}. \tag{7.101}$$

It is further noted that

$$\mathbf{ln}_{-n}\, x = \left(\mathbf{ln}_n\, x\right)^{-1} \tag{7.102}$$

For all integer values of $n > 0$, we redefine (7.85) and have

$$y = nU_n = n\,\mathbf{ln}_0\, x\left(x^n\right) \tag{7.103}$$

$$nx^n = \frac{nU_n}{\mathbf{ln}_0\, x} = \frac{y}{\mathbf{ln}_0\, x} \tag{7.104}$$

$$x^n = \frac{U_n}{\mathbf{ln}_0\, x} = \frac{y}{n\,\mathbf{ln}_0\, x} \tag{7.105}$$

and

$$\mathbf{ln}_0\, x = \frac{U_n}{x^n} = \frac{y}{nx^n} \tag{7.106}$$

Multiplying (7.104) by (7.101), gives

$$D_x\left(x^n\right) = nx^{n-1} = \left(\frac{1}{x}\right)nx^n = \frac{nU_n}{\mathbf{ln}_0\, x}\left(\frac{e_{-n}{}^{n} d\,\mathbf{ln}_n\, x}{dx}\right) = \frac{y}{\mathbf{ln}_0\, x}\left(\frac{e_{-n}{}^{n} d\,\mathbf{ln}_n\, x}{dx}\right) \tag{7.107}$$

and it can be seen that the terms on the right are equal to the derivative of x^n.

With some rearrangement we have

$$x^n\left(n\frac{dx}{x}\right) = \frac{U_n}{\mathbf{ln}_0\, x}\left(ne_{-n}{}^{n} d\,\mathbf{ln}_n\, x\right) = \frac{y}{n\,\mathbf{ln}_0\, x^n}\left(ne_{-n}{}^{n} d\,\mathbf{ln}_n\, x\right). \tag{7.108}$$

Since the terms in brackets in (7.108) are equal, it is apparent that the differential of any variable x of order n, of any function $f(x^n)$, is the product of that function and

$$n\frac{dx}{x} = nd\,\mathbf{ln}_0\, x = ne_{-n}{}^{n} d\,\mathbf{ln}_n\, x = W_0\left(n\right)d\,\mathbf{ln}_n\, x \tag{7.109}$$

where the term $ne_{-n}{}^{n}$ is equal to $W_0(n)$ and to the principal branch value of the Lambert W function for n. It follows that the derivative of x with respect to the nth natural log is

$$\frac{dx}{d\,\mathbf{ln}_n\, x} = \frac{xd\,\mathbf{ln}_0\, x}{d\,\mathbf{ln}_n\, x} = e_{-n}{}^{n} x = \frac{W_0\left(n\right)}{n} x \tag{7.110}$$

which normalized to the nth power would be

$$1_n = \frac{dx}{xd\,\mathbf{ln}_n\, x} = \frac{d\,\mathbf{ln}_0\, x}{d\,\mathbf{ln}_n\, x} = e_{-n}{}^{n} = \frac{W_0\left(n\right)}{n} \tag{7.111}$$

A factor for normalizing to the 0th power, therefore, would be, $e_n{}^{n}$, remembering (7.93), giving

$$1_0 = e_n{}^{n} \frac{dx}{xd\,\mathbf{ln}_n\, x} = e_n{}^{n} \frac{d\,\mathbf{ln}_0\, x}{d\,\mathbf{ln}_n\, x} = e_n{}^{n} e_{-n}{}^{n} = e_n{}^{n} \frac{W_0\left(n\right)}{n} = \frac{W_0\left(n\right)}{n}\left(e_0{}^{W_0(n)}\right). \tag{7.112}$$

where the last term is of the form of (7.75).

For a negative derivative

$$-1_n = \frac{-dx}{xd\,\mathbf{ln}_n\,x} = \frac{-d\,\mathbf{ln}_0\,x}{d\,\mathbf{ln}_n\,x} = -e_{-n}{}^n = \frac{W_0(-n)}{n} \tag{7.113}$$

the factor is - $e_n{}^n$, so that

$$\begin{aligned} 1_0 &= -e_n{}^n \frac{-dx}{xd\,\mathbf{ln}_n\,x} = -e_n{}^n \frac{-d\,\mathbf{ln}_0\,x}{d\,\mathbf{ln}_n\,x} = -e_n{}^n\left(-e_{-n}{}^n\right) \\ &= -e_n{}^n \frac{W_0(-n)}{n} = \frac{W_0(-n)}{n}\left(-e_0{}^{W_0(n)}\right) \end{aligned} \tag{7.114}$$

If we now introduce a rotational (imaginary) element into this condition, from (7.86) and the above development, we have

$$e_n{}^i = e_0{}^{ie_{-n}{}^n} = e_0{}^{\frac{W(in)}{n}} \;\therefore e_n{}^{in} = e_{in}{}^n = e_0{}^{ine_{-n}{}^n} = e_0{}^{W(in)} \tag{7.115}$$

$$e_n{}^{-i} = e_0{}^{-ie_{-n}{}^n} = e_0{}^{\frac{W(-in)}{n}} \;\therefore e_n{}^{-in} = e_{-in}{}^n = e_0{}^{-ine_{-n}{}^n} = e_0{}^{W(-in)} \tag{7.116}$$

in which (7.115) and (7.116) are complex conjugates, each representing a unit vector in the complex plane, so that

$$e_n{}^{in} e_n{}^{-in} = 1 + i0\,. \tag{7.117}$$

It follows that

$$i1_n = \frac{idx}{xd\,\mathbf{ln}_n\,x} = \frac{id\,\mathbf{ln}_0\,x}{d\,\mathbf{ln}_n\,x} = ie_{-n}{}^n = \frac{W_0(in)}{n}\,. \tag{7.118}$$

implying

$$iW(n) = W(in)\,. \tag{7.119}$$

Thus with substitution from (7.96), using the normalizing factor -$ie_n{}^n$, (7.118) becomes

$$\begin{aligned} 1_0 &= -ie_n{}^n \frac{idx}{xd\,\mathbf{ln}_n\,x} = -ie_n{}^n \frac{id\,\mathbf{ln}_0\,x}{d\,\mathbf{ln}_n\,x} = \left[-ie_n{}^n ie_{-n}{}^n\right] \\ &= -ie_n{}^n \frac{W_0(in)}{n} = \frac{W_0(in)}{n}\left(-ie_0{}^{W_0(n)}\right) \end{aligned} \tag{7.120}$$

Similarly for a clockwise rotation,

$$-i1_n = \frac{-idx}{xd\,\mathbf{ln}_n\,x} = \frac{-id\,\mathbf{ln}_0\,x}{d\,\mathbf{ln}_n\,x} = -ie_{-n}{}^n = \frac{W_0(-in)}{n} \tag{7.121}$$

we have the normalizing factor, $ie_n{}^n$, and

$$\begin{aligned} 1_0 &= ie_n{}^n \frac{-idx}{xd\,\mathbf{ln}_n\,x} = ie_n{}^n \frac{-id\,\mathbf{ln}_0\,x}{d\,\mathbf{ln}_n\,x} = \left[ie_n{}^n\left(-ie_{-n}{}^n\right)\right] \\ &= ie_n{}^n \frac{W_0(-in)}{n} = \frac{W_0(-in)}{n}\left(ie_0{}^{W_0(n)}\right) \end{aligned} \tag{7.122}$$

In the above treatment of (7.118) through (7.122), we have used real normalizing factors with imaginary sense. It is further noted that the normalizations shown in the square brackets of (7.120) and (7.122), where the left $\mp e_n{}^n$ operates on the right initial condition, $\pm e_{-n}{}^n$, are instances of a type of complex inversion and are therefore conformal and are not instances of complex conjugation as shown in (7.117). In this latter case we interpret the subscript as the real exponent, *n*, of the *n*th exponential base and the superscript as the rotational or "imaginary" exponent. In this regards we will find that

$$e_n{}^{in} = e_{in}{}^n \tag{7.123}$$

so the determining indication for the rotational exponent is the presence of the *i* sense. Thus we have the following complex identities

$$e_n{}^{in} \equiv e_{-n}{}^{-in} \equiv e_{in}{}^n \equiv e_{-in}{}^{-n} = \overline{e_n{}^{-in}} \equiv \overline{e_{-n}{}^{in}} \equiv \overline{e_{in}{}^{-n}} \equiv \overline{e_{-in}{}^n} \tag{7.124}$$

where the terms on the right are the complex conjugates of those on the left, all of which represent unit vectors whose points lie on the unit circle. Thus it is not to be interpreted in the usual sense of a decomposed complex number, since

$$z_e = e^n + e^{in} \neq e_n{}^{in} = \left(e^n\right)^{in} \equiv \left(e^{in}\right)^n. \tag{7.125}$$

We might also surmise that the above normalizations are analytic. The normalizing factor inverts first with respect to sense of the *n*th degree of the exponential base, e_{-n}, on the unit circle, which amplifies the modulus or vector length of that base. This changes (7.117) to

$$e_n{}^{in} e_{-n}{}^{in} = e_0{}^{in} = 1 + i\,\mathbf{sin}\,\theta. \tag{7.126}$$

This is followed by inversion in the real axis to get

$$e_0{}^{-in} e_n{}^{in} e_{-n}{}^{in} = e_0{}^{-in} e_0{}^{in} = e_0{}^0 = 1 + i\,\mathbf{sin}\,\theta + \left(0 - i\,\mathbf{sin}\,\theta\right) = 1 + i0. \tag{7.127}$$

where it is clear that the 0^{th} exponential base raised to the 0^{th} power is a unit vector on the real, *x* axis. Thus complex normalization in this instance amounts to

$$e_{-n}{}^{in} e_n{}^{-in} = e_0{}^0 = 1 + i0 \tag{7.128}$$

and complex inversion to

$$\frac{1+i0}{e_{-n}{}^{in}} = \frac{e_0{}^0}{e_{-n}{}^{in}} = e_n{}^{-in} \tag{7.129}$$

which is an amplitwist as defined by Tristan Needham in <u>Visual Complex Analysis</u>. This is the case of the bracketed term of (7.120), which might be represented by multiplication of a point in the counterclockwise interior of the unit circle by its reflection in that circle followed by multiplication of the resulting vector and its complex conjugate. In the case of (7.122), we have such multiplication of a clockwise interior vector by a counterclockwise exterior vector, both cases resulting in a unit vector along the real axis.

With respect to (7.89), we see that what appears at first glance to be a singularity is in fact an identity of the 0^{th} order. Remembering that the natural log function maps to the *y* axis and is therefore equivalent to the imaginary axis in the complex plane and $i\,\mathbf{sin}\,\theta$, using the normalization factor, $e_0{}^0 = 1$, and recalling that $x_n = e_n$

$$e_0{}^{+i0}\,\mathbf{ln}\,x_0 = e_0{}^{-i0} = \frac{W_0(0)}{0}e^{W(0)} = x_0{}^{+i0}x_0{}^{-i0} = 1 \pm i0\,. \qquad (7.130)$$

The sense of the 0th powers can be seen as a vector potential or direction, similar to the assignment of charge sense in a static electrical or potential field.

Investigation will show that for any *n* or *q*, real or imaginary,

$$\left(\frac{dx}{xd\,\mathbf{ln}_0\,x}\right) = \frac{W(n)}{n}e^{W(n)} = \frac{W(q)}{q}e^{W(q)} \qquad (7.131)$$

Applying the above normalization factors to (7.83), we have

$$\begin{aligned} -ie_0{}^{W(n)}\frac{dy}{dx} &= \left(-ie_0{}^{W(n)} + W(n)e_0{}^{W(n)}\right)nx^{n-1} = \left(-ie_n{}^n + n\right)nx^{n-1} \\ -ie_0{}^{W(n)}\frac{dy}{d\,\mathbf{ln}\,x} &= \left(-ie_0{}^{W(n)} + W(n)e_0{}^{W(n)}\right)nx^{n} = \left(-ie_n{}^n + n\right)nx^{n} \end{aligned}\,. \qquad (7.132)$$

The interpretation of this development is that while the q^{th} exponential base to the *q*th power maps the real number line *x* to the positive real number line *y*, the q^{th} exponential base to the *iq*th power or the iq^{th} exponential base to the *q*th power maps the real number line to the unit circle. Further, whereas there is an asymptote for the former in the direction of the negative *x* axis, the unit circle is both <u>asymptote and tangent</u> in either sense for the rotational mapping of *e*.

With respect to the integer orders of *e*, it is apparent that each represents a mapping to the real number line of an exponential change in *n* orthogonal spaces. Thus referring to (7.88) in the context of (7.112), we can state the following normalizations of an *n* dimensional *t* or *x*

$$\begin{aligned} 1 = e_n\omega_n &= \frac{e_n}{t_n} = \left(\frac{W_0(n)}{n}\right)^{\frac{1}{n}}\left(e_n{}^n\right)^{\frac{1}{n}} \quad \text{and} \\ &= e_n\left(\mathbf{ln}\,t_n\right)^{\frac{1}{n}} \end{aligned} \qquad (7.133)$$

$$\begin{aligned} 1 = e_n\kappa_n &= \frac{e_n}{x_n} = \left(\frac{W_0(n)}{n}\right)^{\frac{1}{n}}\left(e_n{}^n\right)^{\frac{1}{n}} \\ &= e_n\left(\mathbf{ln}\,x_n\right)^{\frac{1}{n}} \end{aligned}\,. \qquad (7.134)$$

Fleshing this out for the first 4 orders of *n* with conjectured generalization at infinity, we have the following table, as generated by Maple, where it can be seen that a negative *n* is simply an inversion of $e_n{}^n$ to $e_n{}^{-n} = e_{-n}{}^n$.

$f(n)$	n	0	1	2	3	…	∞
e_n		2.718281828..	1.763222834..	1.531584394..	1.419024454..		1
$e_{-n}=e_n^{-1}$		0.367879441..	0.567143291..	0.652918640..	0.704709490..		1
$e_n^{\ n}$		1	1.763222834..	2.345750756..	2.857390779..		∞
$e_{-n}^{\ n}= e_n^{\ -n}$		1	0.567143291..	0.426302751..	0.349969632..		0
$\ln_0 e_n$		1	0.567143291..	0.426302751..	0.349969632..		0
$\ln_0 e_{-n}$		-1	-0.567143291..	-0.426302751..	-0.349969632..		0
$\ln_n e_n$		1	1	1	1		1
$\ln_n e_{-n}$		-1	-1	-1	-1		-1
$\ln_0 e_n^{\ n} = W(n)$		0	0.567143291..	0.852605502..	1.049908893..		∞
$\ln_0 e_{-n}^{\ n} = W(-n)$		0	-0.567143291..	-0.852605502..	-1.049908893..		$-\infty$
$\ln_n e_n^{\ n}$		0	1	2	3		∞
$\ln_n e_{-n}^{\ n}$		0	-1	-2	-3		$-\infty$

n	$f(n)$	$n \ln_0 e_n$	$in \ln_0 e_n$	$n \ln_0 ie_n$	$in \ln_0 ie_n$
1		0.5671…	i0.5671…	0.5671…$+i\pi/2(=+1\ i\pi/2)$	$-\ \pi/2 +$ i0.5671…
2		0.8526…	i0.8526…	0.8526…$+i\ \pi(=+2\ i\pi/2)$	$-\ \pi +$ i0.8526…
3		1.0499…	i1.0499…	1.0499…$+i3\pi/2(=+3\ i\pi/2)$	$-\ 3\pi/2 +$ i1.0499…
4		1.2021…	i1.2021…	1.2021…$+i\ 2\pi(=+4\ i\pi/2)$	$-2\pi +$ i1.2021…
5		1.3067…	i1.3067…	1.3067…$+i5\pi/2(=+5\ i\pi/2)$	$-\ 5\pi/2 +$ i1.3067…
6		1.4324…	i1.4324…	1.4324…$+i\ 3\pi(=+6\ i\pi/2)$	$-3\pi +$ i1.4324…

In this final table, it is clear that the integers, n, are the count of the rotations of ½ π and of the powers and hence the number of orders of i, both indications of a degree of orthogonal structure.

The special case of e_2 is shown to be of fundamental significance to an understanding of the foundations of quantum mechanics. Thus

$$\begin{aligned} e_2 &= \left(\mathbf{ln}\, e_2\right)^{-\frac{1}{2}} \\ &= \frac{1}{\omega_2} = \partial t = \left(\mathbf{ln}\,\partial t\right)^{-\frac{1}{2}} \\ &= \frac{1}{\kappa_2} = \partial x = \left(\mathbf{ln}\,\partial x\right)^{-\frac{1}{2}} \\ &= 1.531584394\mathbf{...} \end{aligned} \tag{7.135}$$

With respect to a conservative 3-field, here shown for a stress, where the volume potential energy density is conserved, a logarithmic change of the tension fields in one dimension, leading to a change of opposite sense in the two shear fields,

$$\left(\mathbf{ln}\,\partial T_{\xi}\right)\left(\partial T_{\eta}\right)\left(\partial T_{\zeta}\right) = E_1 = 1 \tag{7.136}$$

is therefore

$$\partial T_{\eta,\zeta} = \left(\mathbf{ln}\,\partial T_{\xi}\right)^{-\frac{1}{2}} \tag{7.137}$$

which can be stated by using the coefficients as

$$e_2 \partial T_0 = \left(\mathbf{ln}\, e_2\right)^{-\frac{1}{2}} \partial T_0 \,. \tag{7.138}$$

Assuming the change in the tension field is less than unity results in a negative logarithm and an orthogonal (imaginary) sense to the transverse fields, giving us

$$i e_2 \partial T_0 = \left(\mathbf{ln}\, e_2^{-1}\right)^{-\frac{1}{2}} \partial T_0 = i 1\mathbf{.}531584394\mathbf{...}\partial T_0 \,. \tag{7.139}$$

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