Rest Mass Quantization as a Function of Spacetime Exponential Expansion Stress

Compactification of Time, Geometrization of Quantum Mass and Gravity, and the Fundamental Quantum Metric

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Abstract

This analysis provides a geometric model, capable of being visualized in three spatial dimensions, of rest mass quantization as an emergent property of a classical spacetime continuum by way of a fundamental, locally discrete rotational oscillation that is a function of the exponential expansion stress of that spacetime. A non-Minkowski spacetime is developed in which time is modeled as a local, compactified dimension exhibiting Lorentz covariance and in which fundamental quantum rest mass, m_0, and spin energy, E_0, is a measure of the angular wave number, κ_0, and angular frequency, ω_0, of a resonant oscillation. Quantum gravity, dG_0, arises naturally as the quantum differential of the transverse wave force of this oscillation with respect to a change in spacetime expansion stress, dT_0. The Planck area, $d\mathrm{A}_0$, is shown to be the differential of a fundamental unit area with respect to that change in expansion stress. The strong interaction is the operation of that wave force between two or more quanta within a shared, local wave force domain. This quantum state is expressed as a modification of a

chargeless extreme Kerr metric with an oscillation at resonant frequency of the ϕ coordinates imposed by continuity conditions which prevent coordinate entanglement. Such oscillation results in a rotation of the wave phase at the same frequency. It thereby describes a physical spinor, constituting the quantum magnetic field and the property of ½ spin, and isospin in the presence of other quanta. The ergosphere of this quantum metric is the wave force domain of the strong interaction. From a universal bookkeeper reference frame, the fundamental quantum scale is the neutron scale given by the neutron reduced Compton wavelength. Finally, the analysis indicates that cosmic expansion is accelerating exponentially from a condition of maximum density, is presumably cyclical and that in terms of the current time scale, it is approximately 285 billion years into the current expansion cycle. General relativity requires the following refinement in this model; spacetime acquires the property of inertial density as a potential energy density independent of any energy or rest mass quanta, has an exponential expansion rate, and admits torsion that prevents the orientation entanglement condition.

1 – Kinematics and the Geometrization of Time

> "Mechanics . . . is generally regarded as consisting of kinematics and dynamics. Kinematics . . . is the science that deals with the motions of bodies or particles without any regard to the causes of these motions. Studying the positions of bodies as a function of time, kinematics can be conceived as a space-time geometry of motions, the fundamental notions of which are the concepts of length and time. By contrast, dynamics, . . . is the science that studies the motions of

bodies as the result of causative interactions. As it is the task of dynamics to explain the motions described by kinematics, dynamics requires concepts additional to those used in kinematics, for “to explain” goes beyond “to describe”.” [1]

To take up the task set forth by Max Jammer, we might look for explanation of dynamics in a greater understanding of those “concepts additional”, chief of which is mass; in particular we might seek “to explain” mass through a more detailed description of the kinematic concepts of length and time. We would seek to find a definition of mass as a measure of length and/or time. In order to properly undertake such an investigation, we must first examine the concepts of length and time.

Length is a concept used to quantify the apparent spatial separation of entities, where entity might be any distinction within the field of observation, including the two ends of a rod. It is of interest that the magnitude of time is also referred to as a length. We easily conflate measures of separation in time and in space with one term, length, and to contrast them as a ratio, speed. However, there is no more than a conventional preference for the ordering of that relationship, as a mile in four minutes and a four minute mile despite a numerical difference indicate the same physical change, the race speed or

$$c_{race} = \frac{1\text{ mile}}{4\text{ minutes}} = \frac{4\text{ minutes}}{1\text{ mile}} = \frac{1\text{ space or time interval}}{1\text{ time or space interval}}. \qquad (0.1)$$

In a similar fashion, we can state a number of times per time or of lengths per length, i.e. a frequency in time or space, as

$$f_t = \frac{4 \text{ flashes}}{1 \text{ second}}\text{, or } f_l = \frac{3 \text{ feet}}{1 \text{ yard}}\,. \tag{0.2}$$

A length of spatial or temporal separation can be termed an interval between entities or events, as in general relativity. A single entity can have multiple events, as with a flashing beacon, and a single event can have multiple entities, as with a "big bang", as well as multiple perceptions of the event. This does not mean that time and distance are the same qualities by virtue of the use of this common reference term, but it suggests we might equate them mathematically with some universally acknowledged gauge. Thus the speed of light in vacuo, held to be a maximum, is used to gauge a length of time, converting it to a length of distance. We might also use as our gauge some minimum, for example the Hubble rate, approximately 7.87 x 10^{-27} times smaller than the speed of light.

The use of the same term for a separation by time and by space can be misleading. Spatial length is a primary concept, understood by common experience. In simplest manner, its magnitude is determined by holding two objects in proximity, one of which is a standard and the other of which is a test object. We might also consider temporal length as a primary concept, however, we tend to define time <u>quantitatively</u> in terms of a primary spatial length component of an otherwise cyclic or periodic concept, as a comparison of the length rate of change along the circumference of a clock face contemporaneous with some other change.

Taking a hint from the nomenclature of simple arithmetic, we state that a velocity is some translational displacement divided by the number of <u>times</u> some cyclic distance is transited at a constant rate, i.e. the number of times a clock hand tip transits a circumferential distance on the clock face designated as a unit standard interval. In the final analysis velocity is a comparison of two physical lengths, where the customary practical human standard is gauged to correspond with the tangential distance the earth rotates at the equator during (approximately) 1/86,400th of its diurnal cycle, i.e. a second.

The reader may object that it is not the length transited, but the angular speed that marks out time, pointing to the cyclical property with which it is customarily endowed. For a fixed reference frame, all 60 second analog clocks move ideally at the same angular rate, resulting in a varied velocity at hand tip that is a function of the hand length. We might envision that this velocity is limited by the speed of light, and for an ideal clock we stipulate that the length of time taken for light to travel from the center of the clock face to the end of the hand, be it hour, minute, second, nanosecond or yoctosecond, is equal to the length of time for the tip of the hand to travel the same distance tangentially about the face for one radian. Thus its angular frequency, ω, will be inversely related and gauged to the length of its arm, r, or abstractly to an angular wave length, $\lambdabar$, and consequently directly related by the angular wave number, κ, by the constant velocity, c, given by the familiar relationships

$$\omega = \frac{d\theta}{dt} = \frac{c}{r} = \frac{c}{\lambdabar} = c\kappa \, . \tag{0.3}$$

Some rearrangement and integration of the angular measure, using a normalized value for the speed, c = 1, gives

$$r\int_0^1 d\theta = c\int_0^{\left(\omega^{-1}\right)} dt, \quad \therefore |r| = |t| \tag{0.4}$$

If we treat r as a 3-vector, $\boldsymbol{r}$, (calling the clock arm $\boldsymbol{r}$), its origin at the center and its extension point at the circumference of the clock, it is clear that $d\theta$ is orthogonal to $\boldsymbol{r}$. The unit integral of $d\theta$, along with the orthogonal sense, i, is thus an operator that transforms $\boldsymbol{r}$ orthogonally into an instant tangent vector, $c\boldsymbol{t}$, that carries the tip of $\boldsymbol{r}$ with it, rotating $\boldsymbol{r}$ about its origin as

$$\boldsymbol{r} = ic\boldsymbol{t} \qquad (0.5)$$

for which the scalar form, leaving the i for emphasis, is

$$r = ict\ . \qquad (0.6)$$

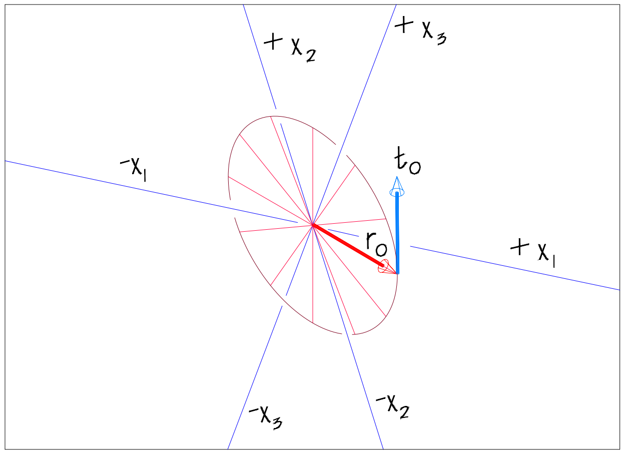
Such orthogonality is what a dimensional relationship between space and time demands. The c is simply a reminder that r and t are normalized, and can be left out by using the ought subscript to indicate unit values in

$$\boldsymbol{r}_0 \equiv i\boldsymbol{t}_0\ . \qquad (0.7)$$

Since $\boldsymbol{r}$ is radial and $\boldsymbol{t}$ is tangential, it is immediately apparent that in addition to being orthogonal to a spatial length, $\boldsymbol{r}$, time is locally cyclical. After a period of 2π it will return to its starting point and continue to cycle at the invariant rate or angular frequency

$$\omega_0 = \frac{d\theta}{dt} = \frac{c}{r_0}\ . \qquad (0.8)$$

We can rotate and translate $\boldsymbol{r}_0$ to any direction and place in three dimensional space, and $\boldsymbol{t}_0$ will remain extended orthogonally from the instant point of $\boldsymbol{r}_0$, as in Time Scale 1.



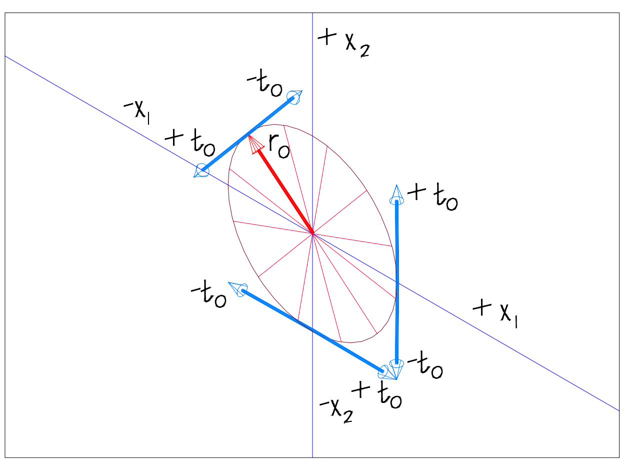
Clock Face Fixed with Rotating Hand
Time Scale 1
Figure 1

We can equate the instant $\boldsymbol{r}_0$ to a unit base vector along an instant spatial dimension x_1, for which x_2 and x_3 are the remaining instant orthogonal dimensions. Since we are limited to three spatial dimensions, $x_{i=1,2,3}$, in most graphic representations the addition of an orthogonal linear dimension of time, $t = x_0$, involves representational difficulty. If we shift the origin of $\boldsymbol{t}_0$ to the origin of the vector $\boldsymbol{r}_0$, so that $\boldsymbol{t}_0$ is co-linear with another unit vector along x_2, call it $i\boldsymbol{r}_0$, we have a 2 dimensional graphic representation of spacetime by substituting the dimension x_0 for x_2. In a 3 dimensional depiction, we can make the equation of $x_0 = x_3$, representing space as a two dimensional plane, x_1-x_2. Both methods are used in discussions of general relativity, with the familiar warping of spacetime represented by a curving funnel in the 3-D depiction. These representations essentially depict time as a linear dimension substituted for one of the suppressed spatial ones.

While such representation has its time tested merits, it yet depends upon the explicit relationship of equation (0.5), which in turn retains the implicit relationship of equation

(0.8). We would hope to find a representation of spacetime which can depict time explicitly as orthogonal to all three dimensions of space, without the suppression of one or two spatial dimensions. In such case, time is thought of as a compactified dimension resident on some local scale, r_0, at each locus of 3-D space.

For such a registration of time, instead of a hand moving about a clock face, we might imagine the entire transparent face rotating about some center. Any spot on the circumference at a distance of r_0 from the center represents the origin of a tangent unit time vector $\boldsymbol{t}_0$, its direction either clockwise or counterclockwise depending on which side of the face one is viewing. The clock face, i.e. time itself, then is moving orthogonal to two spatial dimensions, say x_1 and x_2, as shown in Time Scale 2. Note that the face is moving orthogonal to any instant $\boldsymbol{r}_0$ superimposed upon it and to any arbitrary x_1 and x_2 coordinates centered on the origin of $\boldsymbol{r}_0$.



Clock Face Rotates with Hand
Time Scale 2
Figure 2

Since we have stipulated above that the clock hand can be rotated or translated without changing the relationship of equation (0.5), the same can be said for a rotation or translation both in 2-D and in 3-D space of the whole clock face. Sticking to the 2-D case, in x_1-x_2, we can designate a pair of differential vectors, $\boldsymbol{dt}$, pointing clockwise and counterclockwise, at each possible location of the point of an $\boldsymbol{r}_0$ about the clock face, so that the sum of all $\boldsymbol{dt}$ forms two superimposed circles about the instant center of the clock face. The dimension of time then forms a circle of radius r_0 about each point in x_1-x_2. This can be related to a polar coordinate system, in which the arm of the clock face, $\boldsymbol{r}_0$, is a norm and the x_1-x_2 plane is sectioned as the θ coordinate about its origin.

For a 3-D space, in x_1-x_2-x_3, we can once again designate a differential vector pair, $\boldsymbol{dt}$, at each possible location of the point of an $\boldsymbol{r}_0$ about the clock face and at each possible orientation of the clock face within the 3-space, so that $\boldsymbol{dt}$ can point anywhere in a tangential plane and so that the sum of all $\boldsymbol{dt}$ form a sphere about the instant center of the clock face. Thus the dimension of time, t, is orthogonal to all three spatial dimensions, x_i, of any arbitrary spatial orientation at the points x_i = +/-1.

Now we can simplify and make things a bit more definite as in Time Scale 3. For any clock face θ of radius r_0 in θ, an arbitrary x_1-x_2 plane, rotating about an axis, **θ**, aligned with the x_3 axis orthogonal to x_1-x_2, we can find a second clock face ϕ of equal r_0, concentric with, orthogonal to, and rotating with θ, i.e. spinning like a coin, while itself simultaneously rotating at the same frequency, $\omega_\phi = \omega_\theta$, about an arbitrary axis, **ϕ**, where ϕ rotates in θ and with θ. We can now choose a clock hand, $\boldsymbol{r}_{0,}$ its origin at the center of the concentric clock faces, initially at x_2 at one of the two radial intersections of ϕ and θ, and rotate it with ϕ about **ϕ**, so that

1. at $t(\theta) = 0$, $\boldsymbol{r}_0$ points to (0,+1,0) and $\boldsymbol{t}_{0\phi}$ points to (0,+1,+1);

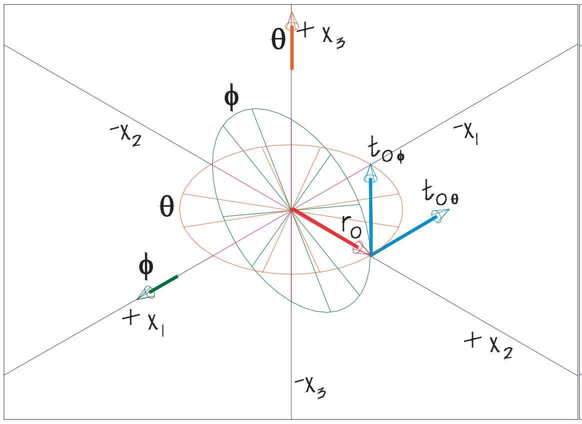
2. at $t(\theta) = \pi/2$, $\boldsymbol{r}_0$ points to (0, 0, +1) and $\boldsymbol{t}_{0\phi}$ points to (+1, 0, +1);

3. at $t(\theta) = \pi$, $\boldsymbol{r}_0$ points to (0,+1,0) and $\boldsymbol{t}_{0\phi}$ points to (0,+1,-1);

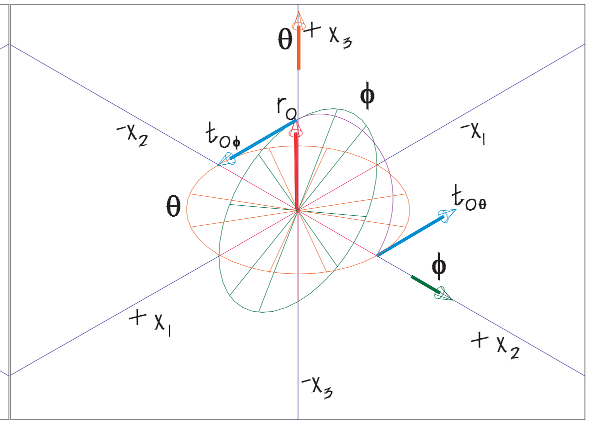
4. at $t(\theta) = 3\pi/2$, $\boldsymbol{r}_0$ points to (0,0,-1) and $\boldsymbol{t}_{0\phi}$ points to (+1,0,-1); and finally

5. at $t(\theta) = 2\pi$, $\boldsymbol{r}_0$ points to (0,+1,0) and $\boldsymbol{t}_{0\phi}$ points to (0,+1,+1);.

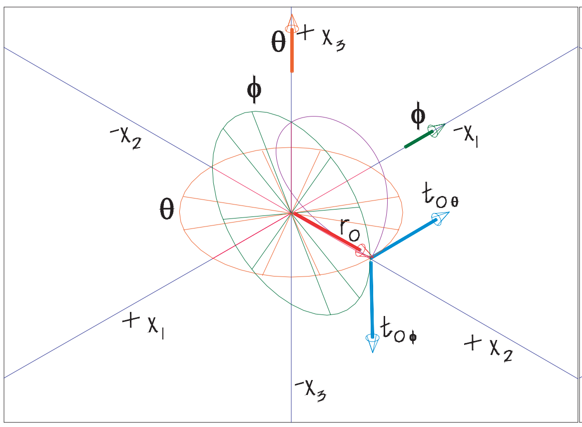
There are an infinite number of $\boldsymbol{r}_0$ in ϕ, they each intersect with the clock face of θ twice and at the same location in θ with each cycle of ϕ about **θ**, and they each extend once to each of the extrema in the ϕ coordinate at $+/-\pi/2$, i.e. at $x_3 = +1,-1$. Thus the point of each $\boldsymbol{r}_0$ and the origin of its time vector $\boldsymbol{t}_0$, traces a figure eight oscillation about one half of the spherical shell formed by the sum of all time vectors ***dt***, while a wave phase rotates counterclockwise about **θ**. Note that his motion avoids the coordinate entanglement condition as depicted in <u>Gravitation</u> by Misner, et al., [2]. We can use this graphic depiction of time to great advantage later in our discussion.



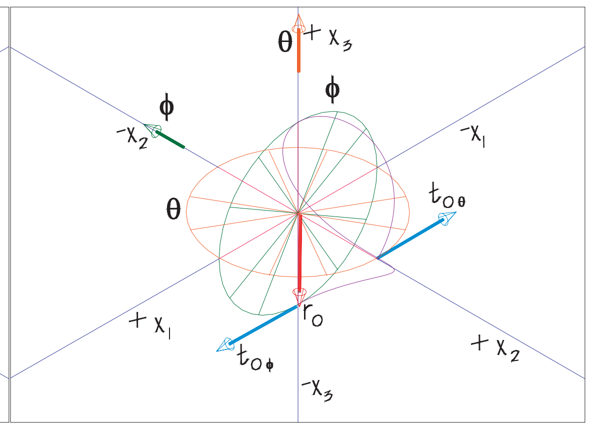
Time t(θ) = 0
Figure 3



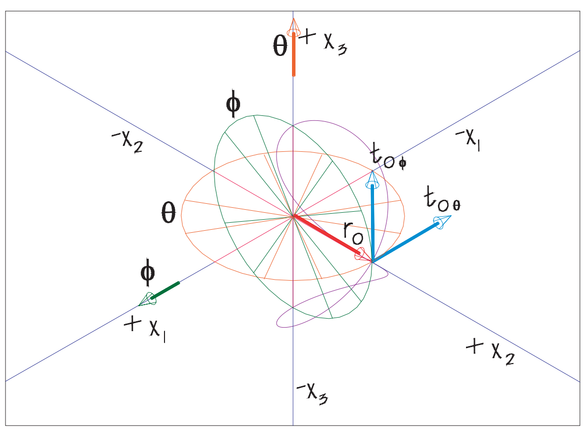
Time t(θ) = π/2
Figure 4



Time t(θ) = π
Figure 5



Time t(θ) = 3π/2
Figure 6



Time t(θ) = 2π
Time Scale 3 - Clock Face Rotates with Hand
and Spins on Edge at Common Frequency
Figure 7

Note that the same instant of time is represented anywhere on the spherical surface of this clock, so that the surface constitutes a time co-ordinate singularity. We can keep track of the "length" of time by a count of the number of oscillations of a given $\boldsymbol{r}_0$.

Finally we can envision that the length of $\boldsymbol{r}_0$ is in some manner augmented or diminished by a very small amount continually with each oscillation, so that the time dimension is seen to be wound up in the manner of a kite string about a constantly increasing or decreasing spatial unit sphere. It is important to remember, however that there are an infinite number of such ***dt*** continually connected in spherical fashion, so the string analogy should not be stretched too far. It is really the expanse of 3-space both about and within such unit sphere, expanding or contracting, that marks the passage of time. It is the expansion of this space at the speed of light, not radialy but tangentially, that gauges time in this spacetime.

Lorentz Covariance

To complete this analysis, we would like to see if this formulation is Lorentz covariant, if the standard of time, t_0, will undergo a scale transformation along with the length standard, r_0, according to the principles of special relativity. Returning to equation (0.6), we might envision that under some condition due to acceleration, such as that of cosmic expansion, r_0 contracts to $r_0^o < r_0$. We divide that equation into its contracted version,

$$\frac{r_0^{\ o}}{r_0} = \frac{ict_0^{\ o}}{ict_0} = \frac{t_0^{\ o}}{t_0} \tag{0.9}$$

and find the unit time standard varies according to the ratio of the unit lengths, as

$$t_0^{\ o} = \frac{r_0^{\ o}}{r_0} t_0 . \tag{0.10}$$

In special relativity as represented by Charles Stevens [3], time intervals transform according to

$$t' = \gamma\left(1-\beta\right)t \tag{0.11}$$

where t is the interval in reference frame F and t' is the same interval viewed in reference frame M moving relative to F at velocity, v, as a fraction of the speed of light, c, giving the ratio identity β, which cannot be greater than 1, as

$$\beta \equiv \frac{v}{c} \tag{0.12}$$

and the value of γ, which cannot be less than 1, as

$$\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} . \tag{0.13}$$

One minus β approaches 0 faster than the inverse of equation (0.13), so the combined factor never exceeds 1 and approaches 0 at the limit. If a relationship can be established between the time dimensions in equations (0.10) and (0.11), then we might expect a relationship between the factors on the right sides. We can do this by viewing a unit standard, t_0, from F and from M.

The spatial interval transformation, in which we have aligned r with an arbitrary x_i axis, is

$$r' = \gamma\left(r - vt\right) \tag{0.14}$$

Substituting from equation (0.6) for t, we have

$$r' = \gamma\left(r - v\frac{r}{c}\right) = \gamma\left(1-\beta\right)r \tag{0.15}$$

which is symmetric with equation (0.11).

Rearranging gives an expression of a proper time, τ, and a proper length, σ, which are invariants of M. We are not using Minkowski space for our 4-vector and r is simply ct, so that multiplying equation (0.16) through by c gives us equation (0.17). This proper length will be shown to be related to r_0^o.

$$\tau \equiv \frac{t'}{\gamma} = \left(1-\beta\right)t \tag{0.16}$$

$$\sigma \equiv \frac{r'}{\gamma} = \left(1-\beta\right)r\ . \tag{0.17}$$

In the Chart 1 graphic representation of a Lorentz transformation we have aligned the spatial axis, r, of a stationary reference frame, F, with the direction of travel of a moving frame, M, making it a pure transformation or boost. This is expressed for the time dimension by equation (0.11) and for the space dimension by equation (0.15). In each equation, the unprimed coordinate with respect to F is modified by the two related factors, (1-β) and γ, to arrive at the primed co-ordinate with respect to M.

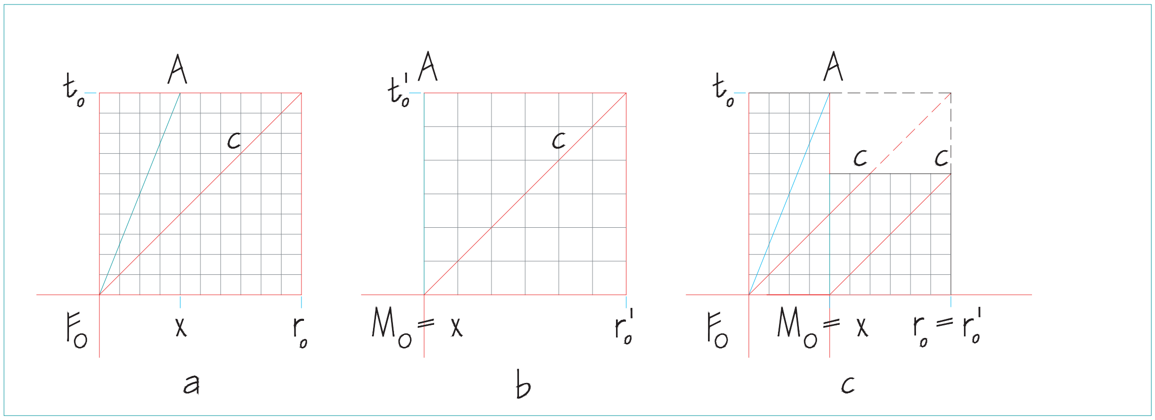


Chart 1 - (1-β) Component of Boost
Figure 8

While the customary analysis is for an arbitrary *x*, or in this case *r* and *t* in *F*, we will apply the same to a space and time unit length in *F*, r_0 and t_0, orthogonally aligned. This is shown in Chart 1.a. The path of *M*, moving at a velocity *v*, or a distance of *x* in time t_0, is drawn as the sloped line, and terminates at point $A = F(x,\ t_0)$. The unit of spacetime has been marked off in decimal fractions. The limit of relative frame velocity, *c*, is shown with its inverted slope of $1\,t_0$ per $1\,r_0$.

In Chart 1.b, the operation of (1-β) on *F* indicates the effect of the motion of *M*, which transforms the unit spacetime from that shown in 1.a. Assume that both *F* and *M* begin to receive a periodic signal from beyond the left edge of their respective charts when those charts are coincident at $r = r' = 0$. They both know that the signal flashes are spaced one-tenth of t_0 apart. As shown, *x* and therefore β happens to be 0.4, resulting in a (1-β) of 0.6. After one t_0, *F* counts ten flash intervals, but *M* has by that time moved four intervals to the right and only counts six intervals. As a result, for *F* the perceived time elapsed before the first signal reaches r_0, therefore the distance from 0 to r_0 is ten-tenths or unity, while for *M* that time and distance is six-tenths of t_0 and r_0 respectively.

Note that the path of *M* observed from *F* in Charts 1.a and 1.c, the diagonal through space and time, is perceived by *M* in his own view of this spacetime, as simply a path through time, shown by the vertical line, $M_O - t_0'$. Note also that the shortening of the time scale is required if *c* is to remain normalized and invariant.

This is not the time dilation and space contraction of relativity, however. If the signal had been coming from the right, during the time t_0, *M* would have counted fourteen intervals to a count of ten for *F*, or a factor of (1+β). This is simply an instance of the Doppler effect, a frequency shift.

As can be seen in Chart 1.c, the gauge or scale factor of the spacetime is the same in both frames, as indicated by the identical grid intervals. The unit time and distance scales of the spacetime for each are not themselves modified by this observed modification, and we will disregard it in the remainder of the discussion. It is of interest, though, that the product of these two factors equals the square of the inverse of the other factor, γ, or

$$(1-\beta)(1+\beta)=(1-\beta^2)=\sqrt{1-\beta^2}^{\,2}=\gamma^{-2}. \tag{0.18}$$

It is the factor γ that we are primarily interested in, as it embodies the change in the scale of spacetime reflected in a measured interval through the Fitzgerald-Lorentz length contraction,

$$\gamma \ \Delta r = \Delta r' \tag{0.19}$$

and through time dilation,

$$\gamma \ \Delta t = \Delta t' . \tag{0.20}$$

These in turn are related to a change in the proper time, τ, and proper length, σ, as in the identity terms of equations (0.16) and (0.17) as

$$\gamma dt = dt' = \gamma d\tau \tag{0.21}$$

$$\gamma dr = dr' = \gamma d\sigma \tag{0.22}$$

Following this line of thought, we substitute the unit standards for the unprimed interval coordinates in equations (0.16) and (0.17) and their contractions for the primed to arrive at an expression of a unit proper time, τ_0, and a unit proper length, σ_0, where each is the representation of the unit standards of *M* in *F*,

$$\gamma \ \tau_0 \equiv t_0^{\ o} = \gamma\left(1-\beta\right)t_0 \tag{0.23}$$

$$\gamma \ \sigma_0 \equiv r_0^{\ o} = \gamma\left(1-\beta\right)r_0 \tag{0.24}$$

Some care is in order here. While the length contraction is often interpreted as a property by which a moving body shrinks absolutely in proportion to its velocity with respect to a stationary frame, and while this may in some instances be true, its fundamental statement is that the unit standard by which a length, *l*, is measured in a moving frame is smaller than the unit standard in the stationary frame with respect to which it is deemed to be moving and from which it is held to be shorter.

In a similar manner, time dilation is deemed to indicate that a given duration of time in a moving frame is measured as moving slower from a stationary frame; thus the usual depiction of the space traveler who returns to earth after 50 years of near speed of light travel, having aged only a couple of earth years. As in the last paragraph, equation (0.20) states the same physical condition as equation (0.19), that the unit standard of time in a moving frame is smaller than the unit standard in a stationary frame, thus an interval of time is measured as greater, i.e. longer as is a length, in the moving frame, but this does not necessarily mean slower.

If our clocks in both the moving and the stationary frame are defined as having hands of a length measured by equation (0.19), and the speed of the end of the hand is the speed of light, c, then the moving frame will have a longer arm and its angular velocity will necessarily be less than that of the stationary frame, and the clock in M will rotate at a slower rate than in F. This is the general interpretation of time dilation. On the other hand, if the length of the hand in M is set to the unit length standard, smaller in M than it is in F, then the speed of light constraint for the speed of the hand tip will result in an increased angular speed and the clock in M will spin faster. In such case time will still be measured as greater, i.e. longer in M than in F, as a count of the number of clock cycles would indicate, in keeping with equation (0.20), since γ in this case is a measure of the relative angular frequencies of *M* and *F*. This is so even though the length of the clock hand path in keeping with equation (0.8) is the same, or

$$r_0\omega_0 = r_0^{\ o}\omega_0^{\ o} = c \tag{0.25}$$

since

$$\frac{r_0}{r_0^{\ o}} = \frac{\omega_0^{\ o}}{\omega_0} = \frac{c}{r_0^{\ o}\omega_0} = \gamma \ . \tag{0.26}$$

With this in mind, we can combine equations (0.20) and (0.19) as we did in equation (0.9), converting from incremental to differential values, and get the equivalent of equation (0.10), where this last case explicitly shows the equivalence of the differential length ratio and γ,

$$dt' = \frac{dr'}{dr}dt = \gamma dt \ . \tag{0.27}$$

We have a temporary conundrum, however, as $\gamma \geq 1$, but the unit length ratios in equation (0.10) and again if inverted from equation (0.26) is less than 1. The problem arises from the nature of a unit standard. If it is fixed, any change in an interval, differential or incremental, will vary directly, proportional to the standard. If the standard itself varies, then the numerical value of a fixed interval will vary indirectly to the change in the standard.

Given a fixed interval, $l \equiv l'$, which is related nominally by γ as measured from frames *M* over *F*, equation (0.27) measures the identical interval, $dt \equiv dt'$ from two different physical standards. Equation (0.10) relates two unit standards, $t_0 > t_0^{\,o}$, that vary proportionally to the two other unit standards, $r_0 > r_0^{\,o}$, all related by *c*. Thus

$$\frac{l}{r_0}\gamma = \frac{l'}{r_0^{\,o}} \tag{0.28}$$

$$\therefore \gamma = \frac{r_0}{r_0^{\,o}} \tag{0.29}$$

We return now to the charts to see how this might be represented graphically. Chart 2 shows an enlargement of the top portion of Chart 1.a in the neighborhood of the time t_0 in *F*. We are analyzing only the effects of the factor γ on the two reference frames and disregarding the Doppler effect of (1-β). Point *A* represents the intersection of the line of motion of *M* in *F* and the time coordinate in *F* for time t_0, designated as $F\left(x, t_0\right)$. In reference frame *M*, based on the above discussion and equations (0.19) and (0.20), this same point would be measured as $M\left(\gamma x, \gamma t_0\right)$, which as drawn for $\beta = 0.4$, so that $\gamma = 1.0910...$, would be (0.4364…, 1.0910…). Finally, based on these same two equations this point is expressed as the intersection of x' and $t_0{}'$, as shown in the square brackets or $M\left(x', t_0{}'\right)$.

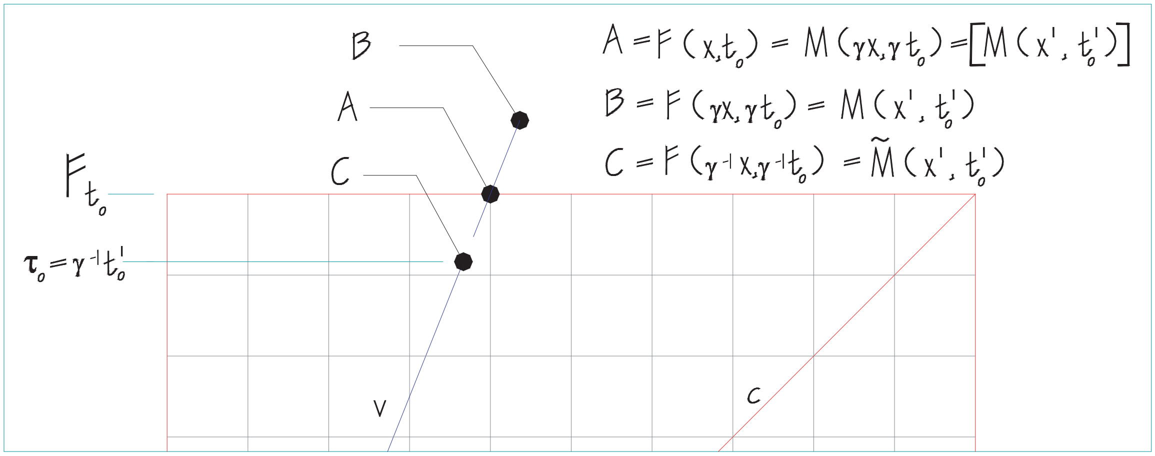


Chart 2 - γ Component of Lorentz Transformation
Figure 9

Point *B* shows the co-ordinates in *F* corresponding to the numerical values of $M(\gamma x, \gamma t_0)$, and therefore represents an expansion of the line for *v* by the value of *γ*. Thus it expresses *M* in terms of *F* and is numerically equal to the value $M(x', t_0')$.

Point *C* shows the numerical value in terms of *F* for the inverse of $M(x', t_0')$ or $\tilde{M}(x', t_0')$. Thus if we were to designate $M(x', t_0')$ in *M* as $M(0.4, 1.0)$, $\tilde{M}(x', t_0')$ would be $F(0.366..., 0.916...)$. The time component of *C* then represents the *proper time*, τ_0, the naught subscript used to indicate its specificity to a unit time standard, t_0', of *M*, when measured from *F*, and in keeping with the concept of time dilation, it is longer in *M* than in *F*. Thus for a value in *M* of $t_0' = 1$, *F* will perceive an elapsed time in *M* of $\tau_0 = 0.916...$. Once again, while generally interpreted as a slowing of time in *M*, this "lengthening" of time can be attributed to a shortening of the time standard.

This is all very interesting, but it would be more illustrative if we could find an essential depiction of the relationship of *F* and *M* involving γ and τ_0. For instance, the length of *v* from F_0 to the three points, *A*, *B* and *C*, embodies the factor of γ, but that factor does not arise naturally, or at least readily, from an analysis of the charting of *v*.

The problem lies in the dual utility of the chart itself. On the one hand it represents a Cartesian background for the plotting of two related bits of data, location in time and in space. From this perspective, the right hand end of the speed of light curve, *c*, at the upper right corner of the chart, represents the time elapsed in *F* during the displacement of a light wave or photon by one unit, or $F\left(r_0, t_0\right)$. On the other hand, it is a 2-D chart of spacetime itself, where the speed of light determines the unit speed for the passage of a stationary reference frame through time or of a displacement through space with no passage of time. This second usage means that in time t_0, the limit of travel of a spacetime vector in the unit spacetime is a circle, or in our chart, a quarter circle, described by the unit spacetime vector, R_0, where

$$R_0 = \left(r^2 + t^2\right)^{\frac{1}{2}}. \tag{0.30}$$

We will use the designation R_0 for both the vector and the circle described using it as a radius, dependent on context. When R_0 is orthogonal to the time axis it is a pure space vector and equals r_0 and when orthogonal to the space axis is a pure time vector and equals t_0. It should be mentioned that in a 4-D spacetime, R_0 is an invariant 4-vector, but that it is not the same 4-vector residing in Minkowski space, as generally used in relativity, as the time vector is not subtracted, but rather is added to the three space vectors, as in equation (0.50).

Drawing this condition on the unit spacetime for *F* gives us Chart 3.a, and we notice immediately that the velocity curve used for the moving frame *M* terminates at *A*, beyond the limit imposed by *c*; that is, it violates one of the basic assumptions of relativity. To correct this, in Chart 3.b we draw the velocity curve, v^o, through the intersection of R_0 and x_1 at A^0, as shown in close-up in Chart 4, and find on closer inspection that this corresponds with the time value of τ_0 for x_1. In fact, for any value of $0 < x < r_0$, this condition will be found to hold, which means that the secant of the angle between t_0 and v^o equals γ, or

$$\frac{\overline{OB^o}}{t_0} = \frac{\overline{OA^o}}{\tau_0} = \gamma \, . \tag{0.31}$$

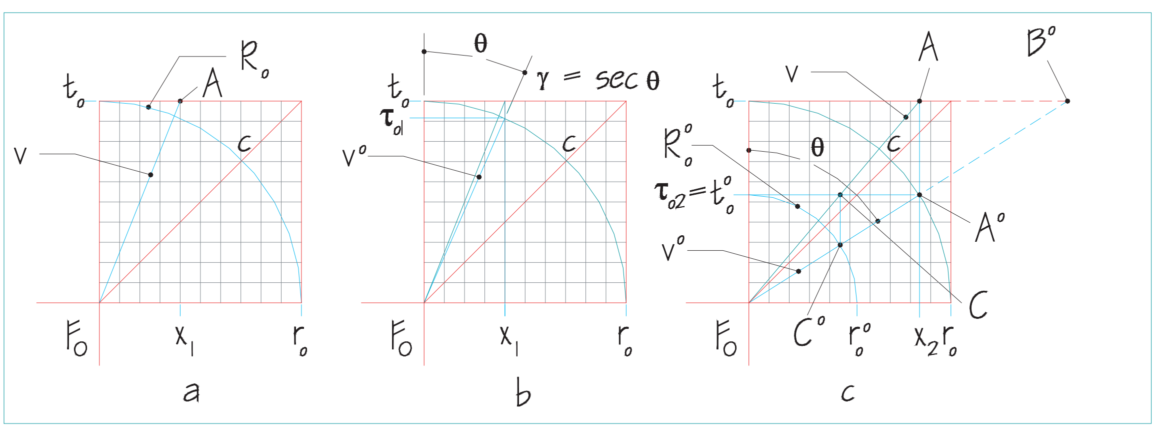


Chart 3 - Contraction of γ Component of Lorentz Transformation
Figure 10

Chart 3.c, a condition at a much higher velocity, shows more clearly the relationship of v^o and τ_0. We construct a second circle for $R_0{}^o$ such that $\left|R_0{}^o\right| = \left|r_0{}^o\right| = \left|t_0{}^o\right|$, where

$$t_0{}^o \equiv \tau_0 \tag{0.32}$$

The orthogonal projection of the intersection of R_0 and v^o onto t_0 intersects at τ_0 and intersects the curve *v* at point *C*, while an orthogonal projection from *C* onto r_0 intersects $R_0{}^o$ and v^o at the same point, C^o. Thus we have the similar triangles, $B^o AA^o \sim A^o CC^o$, and $R_0{}^o$ represents a contraction in the moving frame of the unit spacetime vector R_0.

Chart 4 is a close-up view of the top portion of Chart 3.b, showing the contraction of *v* into v^o. We will call this reference frame F^o. A^o represents the same physical condition as *A* in *F*. The displacement *x* remains as the same percentage of r_0, but the time scale at that point is now the proper time, τ_0. In square brackets, that same point in M^o is numerically identical to $M\left(\gamma x, \gamma t_0\right)$ at *A*, however, the unit time standard, $t_0{}^o = 1$ and the related x^o, are in M^o instead of *F*. Thus the unit standard is local to M^o instead of the expression of a stationary spatial background. If we can think of the time dimension M_t^o as being inclined along the slope of v^o instead of orthogonal as in *F*, the same spacetime numerical values hold in both reference frames.

In keeping with this observation, C^o is proportionally the same to A^o as C is to A. In M^o, the unit time and space standards apply as indicated by $R_0^{\ o}$, so that $M^o\left(x^o, t_0^{\ o}\right)$ is numerically equal to $F\left(x, t_0\right)$ shown at A, or in this case, (0.4, 1.0). For the proper time, with all values in units of F, we have the following identity,

$$\tau_0 \equiv t_0^{\ o} \equiv \gamma^{-1} t_0 \ = \gamma F_t\left(C^o\right) \tag{0.33}$$

Point B^o indicates the nature of time dilation as conventionally figured. At point A^o, M^o has traveled the same length of time as F, as given by $R_0 = t_0$, but to F this is registered as the proper time τ_0. By the time M^o reaches B^o, which F registers as 1.0, F has moved on in time to γt_0. The length of v^o, $\overline{OB^o}$, is equal to γ.

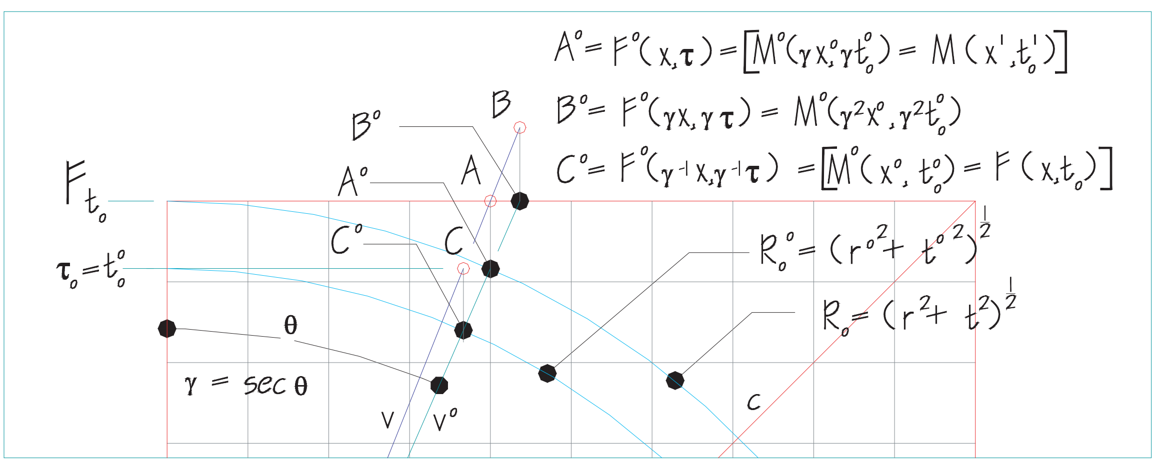


Chart 4 - Closeup of Contraction
Figure 11

Perhaps the most significant aspect of this representation is that the secant of the angle of t_0 O v^o establishes γ, underscoring the geometric nature of time. Expanding on the relationships of equation (0.31), we have

$$\frac{\overline{OB^o}}{t_0}=\frac{\overline{OA^o}}{\tau_0}=\frac{R_0}{R_0{}^o}=\frac{t_0}{\tau_0}=\frac{\overline{OB^o}}{\overline{\overline{OA^o}}}=\frac{\overline{OB}}{\overline{\overline{OA}}}=\frac{\overline{OA}}{\overline{\overline{OC}}}=\gamma \tag{0.34}$$

We have examined the Lorentz transformation with respect to time and proper time, but a similar analysis could be made with respect to space and a proper distance as modified from the conventional Minkowski representation as noted earlier. The case of time is more germane to our present discussion, as will be seen.

The above charts suggest that spacetime curvature is as much a matter of curvature of time as it is of space. Chart 3 indicates that as a moving reference frame approaches the speed of light and v approaches co-linearity with c, v^o and γ approach infinity and co-linearity with the space axis, r, and the time and distance scales indicated by $R_0{}^o$ become exceedingly small, and in the same proportion. This is precisely what we analyzed initially with equation (0.5) and a cyclical time dimension. If we envision M as an accelerating reference frame starting from rest at F_O and accelerating to c within the first unit of spacetime, we would find that the contracted velocity curve, v^o, and the collinear contracted time dimension would curve, and that under certain constraints, would arc like a quarter circle. Its constantly shortening time standard, $t_0{}^o$, then is aligned with the arc of γ, and its space standard, $r_0^{\,o}$, correspondingly shortened, rotates with and orthogonally to it.

A physical instance of this shortening of both $r_0^{\,o}$ and $t_0^{\,o}$ can be found by examining the nature of the deBroglie wavelength of a massive particle. We assume that the reduced Compton wavelength, $\lambda\!\!\!^{-}_C$ is indicative of the rest state of such a particle, and is determined by dividing the reduced Planck's constant, $\hbar$, by the product of the speed of light and the particles rest mass, m,

$$\lambda\!\!\!^{-}_C = \frac{\hbar}{mc}. \tag{0.35}$$

The reduced deBroglie wavelength, $\lambda\!\!\!^{-}_{dB}$, is the quotient of $\hbar$ and the particle's relativistic momentum, p, at velocity, v, given as

$$\lambda\!\!\!^{-}_{dB} = \frac{\hbar}{p} = \frac{\hbar}{\gamma m v} \tag{0.36}$$

where the factor γ is the same as used in the development above. Combining and some rearrangement gives us the ratio of these reduced wavelengths as

$$\frac{\lambda\!\!\!^{-}_C}{\lambda\!\!\!^{-}_{dB}} = \gamma \frac{v}{c} = \gamma\beta = \gamma x. \tag{0.37}$$

In Chart 4, this is represented by the tangent of angle θ between the time axis in F and v^o and gives the ratio of the particle velocity in F, where $A^o(x) = A(x)$, and the contracted unit standard, $r_0^{\,o}$. Thus

$$\frac{\bar{\lambda}_C}{\bar{\lambda}_{dB}} = \frac{x}{r_0{}^o} = \frac{x}{t_0{}^o} = \frac{x}{R_0{}^o} \tag{0.38}$$

and as we approach the limit of the speed of light and x approaches $r_0 = 1$, we have

$$\bar{\lambda}_C = \frac{r_0}{r_0{}^o} \bar{\lambda}_{dB} = \gamma\, \bar{\lambda}_{dB} \tag{0.39}$$

Thus, in the extreme case

$$\text{if } \bar{\lambda}_C = r_0\text{, then } \bar{\lambda}_{dB} = r_0{}^o. \tag{0.40}$$

Since the frequency and wavelength are related as

$$\bar{\lambda}_C \omega_C = \bar{\lambda}_{dB} \omega_{dB} = c \tag{0.41}$$

rearrangement gives, in the extreme case

$$\gamma = \frac{\bar{\lambda}_C}{\bar{\lambda}_{dB}} = \frac{\omega_{dB}}{\omega_C} = \frac{t_C}{t_{dB}} = \frac{r_0}{r_0{}^o} = \frac{\omega_0{}^o}{\omega_0} = \frac{t_0}{t_0{}^o}, \tag{0.42}$$

therefore we also have

$$\text{if } t_C = t_0\text{, then } t_{dB} = t_0{}^o. \tag{0.43}$$

Considering a normalized frequency, that is, where the angular displacement, $\theta = \theta_0$, always equals 1 and the time consequent varies according to the particular F from which it is observed, we can integrate equation (0.25) for any time $t = qt_0 = q^o t_0{}^o$

$$r_0 \omega_0 \int_0^t dt = r_0{}^o \omega_0{}^o \int_0^t dt \tag{0.44}$$

$$r_0 \omega_0 \left(qt_0\right) = r_0{}^o \omega_0{}^o \left(qt_0\right) = r_0{}^o \omega_0{}^o \left(q^o t_0{}^o\right) \tag{0.45}$$

$$qr_0 = q\gamma r_0{}^o = q^o r_0{}^o \tag{0.46}$$

and finally

$$r_0 = \gamma r_0{}^o = \frac{q^o}{q} r_0{}^o \tag{0.47}$$

therefore

$$t_0 = \gamma t_0{}^o = \frac{q^o}{q} t_0{}^o \tag{0.48}$$

showing that γ is simply the frequency ratio of the unit standards of space and time between a moving and a stationary reference frame.

It is worth noting the case when $x = R_0^o = r_0{}^o = \frac{1}{\sqrt{2}} r_0$, that is, when r equals t at the intersection of the curve of c and R_0. If we consider a massive particle as some manner of physical stationary waveform, i.e. a bound, rotating wave, a ratio of r and t of unity represents the point at which the translational displacement of the particle in space begins to outrun the transverse wave displacement, i.e. its displacement in time. It is the point at which the contracted velocity, v^o, equals the speed of light. Prior to that point the waveform would conceivably flatten in space in the form of an oblate spheroid. From that point on, the waveform must contract in all dimensions so that its transverse motion remains in phase with its translation, conforming to the deBroglie wavelength.

It follows immediately that from any reference frame F in 4-D spacetime, for a moving frame M, a unit standard can be given for space by $r_0{}^o$ and for time by $t_0{}^o$, both related to a 4-vector (of additive components), $R_0{}^o$, as

$$R_0{}^o = \frac{1}{\sqrt{2}} \left(\left(r_0^o \right)^2 + \left(t_0^o \right)^2 \right)^{\frac{1}{2}} = \frac{R_0}{\gamma} \tag{0.49}$$

where x_{0i} are 3 orthonormal bases, symmetric with respect to r_0,

$$R_0 = \tfrac{1}{\sqrt{2}}\left(r_0{}^2 + t_0{}^2\right)^{\frac{1}{2}} = \tfrac{1}{\sqrt{2}}\left(\left(\tfrac{1}{\sqrt{3}}x_{01}\right)^2 + \left(\tfrac{1}{\sqrt{3}}x_{02}\right)^2 + \left(\tfrac{1}{\sqrt{3}}x_{03}\right)^2 + t_0{}^2\right)^{\frac{1}{2}}. \qquad (0.50)$$

If we shift the origin of $\boldsymbol{t}_0^o$ in Chart 3.c from the origin of $\boldsymbol{r}_0^o$ to its point, we have the configuration shown in Time Scale 1 and 2. From there we can extrapolate to the 3-D form shown in Time Scale 3 for the expression of a 3 dimensional clock.

A statement is in order concerning the "relativity" of the reference frames *F* and *M*, and that of the spacetime scales R_0 and $R_0{}^o$. Assuming that *F* resides in an expanding 3-manifold, if that residence is isotropic with respect to cosmological red shift, then we can state that the local position of *F* is stationary with respect to space and in an extremal position of change with respect to time. Otherwise, *F* would experience a blue shift in the direction of its travel with respect to space. In similar fashion, *F* could experience such an anisotropy while rotating about a center, perhaps galactic or supergalactic, that is itself stationary or isotropic with respect to cosmological red shift. Thus at every point in spacetime, assuming an isotropic expansion, there exists a potential *F* for which R_0 is a local maximum, though R_0 at all points need not be identical. For any moving reference frame *M* at that same point, $R_0^o < R_0$.

By this analysis, we can envision a 4-D spacetime with Lorentz covariance in which the time dimension is modeled as a compactified rotational dimension orthogonal to the three space dimensions, as developed earlier. Having taken this side-trip into an investigation of spatial and temporal length, we can now look at the concept of mass.

2 – Geometrization of Mass in Classical and Quantum Theories

In his book, Concepts of Mass in Contemporary Physics and Philosophy, Max Jammer delineates three types of mass; inertial, active gravitational (corresponding to a source) and passive gravitational (corresponding to a test particle), and concludes that the jury is still out as to whether these represent distinct concepts of mass. Looking at the related concept of inertia, we can readily see that it can be quantified in terms of length and time by the concepts of linear and angular displacement and their derivatives. For simplicity, we limit our thought experiments to analysis in one spatial dimension, unless stated otherwise.

Inertial Mass

Inertia is a resistance to any change in the momentum of a body:

1. An absolute or infinite inertia would indicate immobility or a displacement of $dx = 0$ from the reference frame of that body or a change in velocity of $dv = 0$ from any arbitrary reference frame, resulting from interaction with another body.
2. An absolute lack of or zero inertia would indicate an instantaneous displacement of *a*, an undefined or relative infinite displacement or change in velocity due to a finite displacement with zero passage of time resulting from such interaction.

3. A finite displacement of a body, *a*, over a finite time duration resulting from its interaction is an inverse measure of its finite inertia, i.e. of its inertial mass.

While "body" has been historically conceived as a classical entity, substitution of the term "particle" understood in quantum terms, should not change the meaning of "inertia". A free body or particle is classically conceptualized as moving within and through a flat, three dimensional space, said space of itself and in the absence of any field potential or other bodies or particles of either mass or energy, constituting both a phenomenological and an ontological void. By the above definition and our expansion of it, however, a space upon which we can superimpose a metric, in and of itself exhibits the property of inertia, since it has a definite resistance to change and in the case of physical space, appears to have a finite, albeit accelerating, expansive momentum as evidenced by cosmological red-shift. By virtue of such property, space even without quantum fields can not be said to be either a phenomenological or an ontological void. Within such space, time can be seen as the path of its inertial change.

In the interest of gaining a geometric, descriptive explanation of mass, inertial or gravitational, we will investigate inertial mass first in a classical target-test body. In general relativity, gravitational field source mass is geometrized in <u>direct</u> relationship to length, and we can find a direct relationship between mass and length in the aggregation of bodies or particles. As in the case of stellar configurations, the product of the volume of the body and its average volume inertial density computes the mass of the body, so that for a given density, the reduced circumference of the body gives a geometrized approximation of its mass. For inertial test body *a* the magnitude of its mass, m_a, is indirectly proportional to the displacement, x_a, over the time interval of an interaction, t_a, under a given impulse, J, from another body or source, which results in a final velocity for *a* of v_a,

$$m_a = \frac{J}{v_a} = \tfrac{1}{2} J \frac{t_a}{x_a} . \tag{1.1}$$

The definition of impulse is the integral of force with respect to time which is equal to a change in momentum, ΔP,

$$\boldsymbol{J} = \int_{t_i}^{t_f} \boldsymbol{F}(t)\, dt = \Delta P . \tag{1.2}$$

While in general the force, hence the acceleration, will vary over the duration of the impulse, for ease of illustration, we will use a constant force and acceleration, i.e., the average over the duration. In this case *a* is accelerated from an initial velocity, v_{ai}, to a final velocity, v_{af}, over the time interval $t_\Delta = t_f - t_i = t_a - t_0 = t_a$. The time subscripts indicate that at time $t_i = t_0$, $v_{ai} = 0$. Starting at the end of such interaction, at time $t_f = t_a$, the final velocity of *a* will be reached at $v_{af} = v_a$, and it will continue on at that velocity as viewed from its original reference frame, *F*.

We assume that the source of the impulse and the test body exist in a steady state in their respective rest frames and in isolation from each other and any other interactions both before and after their collision, but that during their interaction they each undergo an acceleration and an exchange of momentum and energy. Thus the acceleration for *a* is

$$a_a = \frac{v_{af} - v_{ai}}{t_a} = \frac{2x_a}{t_a^2} \tag{1.3}$$

and the force is

$$F = m_a \frac{2x_a}{t_a^2} \tag{1.4}$$

Since the time interval for the acceleration of the body and the time interval found in the statement of its velocity is the same as the interaction interval, t_f, we have the following time independent parameter of the interaction

$$ת = \int_{t_i}^{t_f} \boldsymbol{J}(t)\, dt = \int_{t_i}^{t_f} \boldsymbol{F}(t)\, t_f \;\, dt = \tfrac{1}{2} \boldsymbol{F}\, t_f^2 = m\, \boldsymbol{x}_f \tag{1.5}$$

where the letter ת (tav) is an inertial constant of the interaction, of mass-length dimensions. Equation (1.1) can then be expressed in a time independent scalar form where mass is the inverse measure of the space interval of the interaction,

$$m_a = \frac{ת}{x_a}\,. \tag{1.6}$$

We can postulate a second condition, with J unchanged, for a body b, for which

$$m_b > m_a\,. \tag{1.7}$$

Therefore, we have

$$m_b = \frac{J}{v_b} = \tfrac{1}{2} J \frac{t_b}{x_b} = \frac{ת}{x_b} \tag{1.8}$$

and the following inequality is apparent

$$v_b < v_a\,. \tag{1.9}$$

This suggests that if x_b is equal to or less than x_a,

$$t_b \geq t_a \tag{1.10}$$

and/or if t_b is equal to or greater than t_a,

$$x_b \leq x_a\,, \tag{1.11}$$

but that the time intervals cannot be equal if $x_b = x_a$. However, inequality (1.9) would also hold as long as

$$\frac{\Delta x}{x_a} < \frac{\Delta t}{t_a}\,. \tag{1.12}$$

In any case, the inverse velocity will be greater for v_b, so that if ת is invariant, the mass of the test body is an inverse measure of the displacement and a direct measure of the inverse velocity of the interaction, and a geometrized mass should reflect that kinematic relationship. If the test body is “tethered” in some manner with respect to its initial linear reference frame, so that it is free to move along some circular path into an alternate dimension, that velocity becomes an angular measure instead of a linear one, and mass becomes a measure of the angular frequency.

It is of interest that if we consider a source for our impulse above from a classical body, A, of mass, M_A, where

$$M_A \gg m_a\,, \tag{1.13}$$

moving with an initial velocity of V_A, prior to the interaction with a, we find the interaction conforms to the following equation

$$v_a = \frac{2M_A}{\left(M_A + m_a\right)} V_A\,. \tag{1.14}$$

Therefore, at the extreme, where m_a is negligible,

$$v_a \approx 2V_A \tag{1.15}$$

and the final velocity of the test body is principally a function of the source velocity and not of the source mass. We would expect that a similar relationship would hold, if instead of representing a source in an elastic collision, M_A represented a gravitational source. If gravitational and inertial mass are equivalent, then M_A V_A represents the impulse generated by that source, and the final velocity of a test particle a is limited by equation (1.15). Thus if V_A is limited by the speed of light, c, then v_a will be limited to 2c. While this appears to be a violation of the postulates of relativity, when we examine the properties of an extreme Kerr metric later, we will find some justification, since 2 is the coefficient for the tangential or angular component of the metric at the horizon.

With respect to a quantum interaction, we see that equation (1.5) is related to the action, S, of the interaction, using Maupertuis' principle, by

$$\begin{aligned} S &= \int_{x_i}^{x_f} \boldsymbol{J}\left(x\right)\cdot d\,\boldsymbol{x} = \frac{2m}{t_f}\int_{x_i}^{x_f} \boldsymbol{x}_f \cdot d\,\boldsymbol{x} = \frac{2m\,\boldsymbol{x}_f}{t_f}\cdot\frac{\boldsymbol{x}_f}{2} \\ &= m\,\boldsymbol{x}_f \cdot \frac{2\,\boldsymbol{x}_f}{2t_f} = \sqcap\cdot\frac{\boldsymbol{v}_f}{2} = \sqcap\cdot\boldsymbol{c} = \sqcap\omega\cdot\boldsymbol{x}_f = \hbar \end{aligned} \; . \tag{1.16}$$

As $S = \hbar$ is an invariant of the quantum interaction, and m and x are inversely related, so t must be inversely related to m as well (and directly related to x as demonstrated in the previous section). Inverse time is the expression of a rate or in this case unit frequency of interaction, so that mass is the dynamic representation of the kinematics of that unit or angular frequency of the interaction, which varies in proportion to the mass and with respect to the above relationship of a and b, as

$$\omega_b > \omega_a \, . \tag{1.17}$$

Equation (1.16) indicates that the ratio of mass to frequency is equal to the ratio of the inertial constant and one half the “final” velocity of the particle. If S and $\daleth$ are both invariants, then so must be v_f, and with some substitution and rearrangement we have

$$m = \frac{\daleth}{\frac{1}{2} v_f} \omega = \frac{\daleth}{c} \omega \, , \tag{1.18}$$

where

$$c = \frac{v_f}{2} = \frac{2x_f}{2t_f} = \frac{x_f}{t_f} \tag{1.19}$$

From this we have the following expressions for the impulse,

$$J = \Delta P = 2mc = 2\daleth\omega \tag{1.20}$$

which states mass as frequency in a quantum interpretation, since by multiplying through by $\frac{1}{2} c$, (differentiating with respect to time and integrating with respect to displacement), we have the mass-energy relationship

$$E = \tfrac{1}{2} Jc = mc^2 = \daleth c\omega = \hbar\omega = \frac{\hbar c}{\lambda} \, . \tag{1.21}$$

It follows that

$$\daleth = \frac{\hbar}{c} \, . \tag{1.22}$$

Returning to equation (1.1) and substituting from equation (1.20) gives the following relationship between the length of the interaction and m_a, which is as equation (1.6),

$$m_a = \frac{Jt_a}{2x_a} = \frac{\daleth\omega_a}{c} = \frac{\daleth}{\lambda_a} = \daleth\kappa_a \,. \tag{1.23}$$

We find that for individual quantum mass, i.e. that of the neutron, electron, proton, muon, etc., x_q is equal to the Compton reduced wavelength, $\bar{\lambda}_{C,q}$, for that quantum, as given by

$$x_q = \bar{\lambda}_{C,q} = \frac{\daleth}{m_q} \,. \tag{1.24}$$

Quantum analysis assumes the two fundamental invariants, $\hbar$ and c, to which we have now added an inertial constant, $\daleth$. Some simple numerical analysis applied to the variables of mass, displacement, and time in conjunction with the equations for impulse, the inertial constant, interaction terminal velocity and action will help to clarify the geometric relationship of mass, length and time.

In the following table, Row A gives our initial, normalized condition for the variables valued in brackets in the left-hand column. In the remaining rows of this table we have substituted a new body of the given mass, and assumed different space and time values according to various impulse assumptions. The column on the right states whether the set of assumptions in the variables column violates any of the assumed invariants just stated.

Rows B and C maintain the same impulse and have the same v_f, but the space and time intervals differ and neither maintains the velocity Row A. Row D maintains that velocity, but violates the action and the inertial constant condition. It also departs from the initial value of the impulse. The stipulation of a set value for the impulse was a convenience for purposes of development of our argument, but it is not a necessary or even anticipated condition of a physical interaction. Row E is constrained to maintain that impulse, thus maintaining the velocity found in B and C, but results in a violation of all three invariants and is not a quantum solution.

Only Row F and the related G, while necessarily departing from the initial impulse, avoid a violation of the three invariants. What F and G show at a glance, assuming a quantum context, is that quantum inertial test mass is an inverse measure of space and time, the latter two of which are identically gauged in keeping with the development of the previous section on kinematics in which we saw that $r_0^o = t_0^o = R_{_0}^o$.

In Row F, if the space and time standards are assumed to be smaller by the inverse of the factor γ due to a contraction in a moving frame after impulse, the increase in the mass is found to be by the factor γ, showing that Row F is consistent with the postulates of special relativity. Row G, on the other hand, shows an increase in the space and time standards in keeping with a change in γ and a corresponding decrease in mass as we might find in a moving frame that has decreased its velocity from a prior greater differential with respect to some rest frame.

As a source, mass is a direct measure of the impulse as shown by the second column of Rows F and G. Further, since the space and time intervals are identical, and we might assume symmetrical, i.e. interchangeable, it is apparent that the impulse has the same dimensional form as the spin energy of the quantum. Again, using the angular frequency in computing the final velocity, we have the same form for the inertial constant and that velocity, so that in natural units,

$$c^2 = |\text{ת}c| = |\hbar| \tag{1.25}$$

and mass and frequency measure the same physical condition, interaction per time interval, i.e. the smaller the interaction time, the greater the mass and frequency, as

$$\frac{m}{\omega} = \frac{\hbar}{c^2} = \frac{\text{ת}}{c} = 1. \tag{1.26}$$

	$F(m, x_f, t_f) = 2mx_f t_f^{-2}$	$J = \int_{ti}^{tf} F(t)\,dt = Ft_f$	$\text{ת} = \int_{ti}^{tf} J(t)\,dt = mx_f$	$c = \frac{v_f}{2} = \frac{x_f}{t_f} = x_f \omega_f$	$S(=\hbar) = \int_{xi}^{xf} J(x)\,dx = \text{ת}\frac{v_f}{2}$	Violations of $\text{ת}, c, S$
A	$(1,1,1)$	$2\frac{1(1)}{1^2}(1) = 2$	$1(1) = 1$	$1 = 1(1)$	$1(1) = 1$	
B	$(2, \frac{1}{2}, 1)$	$2\frac{2(\frac{1}{2})}{1^2}(1) = 2$	$2(\frac{1}{2}) = 1$	$\frac{1}{2} = \frac{1}{2}(1)$	$1(\frac{1}{2}) = \frac{1}{2}$	c, S
C	$(2,1,2)$	$2\frac{2(1)}{2^2}(2) = 2$	$2(1) = 2$	$\frac{1}{2} = 1(\frac{1}{2})$	$2(\frac{1}{2}) = 1$	$\text{ת}, c$
D	$(2,1,1)$	$2\frac{2(1)}{1^2}(1) = 4$	$2(1) = 2$	$1 = 1(1)$	$2(1) = 2$	$\text{ת}, S$
E	$(2, \frac{1}{\sqrt{2}}, \sqrt{2})$	$2\frac{2(\frac{1}{\sqrt{2}})}{\sqrt{2}^2}\sqrt{2} = 2$	$2(\frac{1}{\sqrt{2}}) = \sqrt{2}$	$\frac{1}{2} = \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}})$	$\sqrt{2}(\frac{1}{2}) = \frac{1}{\sqrt{2}}$	$\text{ת}, c, S$
F	$(2, \frac{1}{2}, \frac{1}{2})$	$2\frac{2(\frac{1}{2})}{\frac{1}{2}^2}(\frac{1}{2}) = 4$	$2(\frac{1}{2}) = 1$	$1 = \frac{1}{2}(2)$	$1(1) = 1$	None

G	$\left(\tfrac{1}{2},2,2\right)$	$2\dfrac{\tfrac{1}{2}(2)}{2^2}(2)=1$	$\tfrac{1}{2}(2)=1$	1 $=2\left(\tfrac{1}{2}\right)$	$1(1)=1$	None

Table 1 – Numerical Analysis of Invariant Violation of Certain Variable Assumptions

The symmetries are yet more pronounced since the speed of light, written in terms of the properties of a wave, can be stated as the ratio of the angular frequency and angular wave number, κ,

$$c=\frac{\omega}{\kappa} \tag{1.27}$$

which when substituted into equation (1.26) gives us the symmetrical statement for the inertial constant,

$$\text{ת}=\frac{m}{\kappa}\,. \tag{1.28}$$

A couple of words are in order concerning frequency, which are no doubt obvious to reader. First, since it is an expression of the ratio between a count of the number of units or radian contemporaneous with a unit of time, in keeping with the comments concerning equation (0.1), it is equal to a count of one radian per fraction of some larger unit of time. A base or unit frequency would be an extremal, normalized frequency of one radian or other briefest instance of change per one smallest standard of time, $t_0^o=r_0^o=R_{_0}^o$. Thus an interaction of the greatest frequency and therefore greatest energy per equation (1.21) will be the one of shortest duration. Second, while such normalized frequency might be a conventional angular frequency of one radian per unit of time, it might equally be one unit of space per unit of time as

$$c = r_0\omega_0 = r_0 \frac{1}{t_0} = \frac{r_0}{t_0} . \tag{1.29}$$

If we state with respect to Rows F and G that

$$x_f = r_0 \tag{1.30}$$

then the displacement x_f resulting from impulse J, can be a reference to a rotational tangent vector at the circumference of the previously depicted rotating clock, instead of the customary translational displacement vector at or from a point-like particle. Such impulse, under the constraints of an invariant c, results in a contraction of the clock, and a decrease in r_0 and t_0 in keeping with γ, and mass is correspondingly measured as increased. Such impulse could be the result of an inelastic collision with a photon-like source or the acceleration arising from some field potential. It is important to note in regards to a field gradient, that the increase in mass, as with the impulse, can be continual and not in discrete steps and still adhere to equation (1.21), since the frequency can increase continually, while the action, $S = \hbar$, remains invariant at any frequency.

To make graphic sense of this in terms of an inertial spacetime continuum, for modeling purposes we can think of an elastic collision between two bodies of equal mass and spherical shape, a and a', constrained so that their motion oscillates in simple harmonic motion. Next we consider a, instead of being a body or particle, to be a 3 dimensional continuum, non-particulate in composition, isotropic but for a boundary plane at the point of impact from a', where a' is moving normal to and in the direction of a. Instead of the mass quantity of body a, continuum a has a linear inertial density, λ_a. That density is subject to variability and elastic strain in addition to being inertial, so that as a' meets a, the inertial density immediately in front of the line of travel of a' increases, slowing its velocity, and the continuum around the area of impact of a' is strained and curved inward.

If we had assumed an <u>inelastic</u> collision, at some point the momentum of a' would be absorbed by a, which would then remain in a distorted condition, marked by a finite degree of strain and curvature of the continuum around the area of impact, and the impulse would continue to diminish indefinitely into the interior of a.

Given a fixed initial momentum of a', the greater the inertial density and therefore the mass of a, the smaller will be the penetration of a' and the radius of the strain at the area of the impact. We can envision that there are two instances of curving, one as a generally deformed hemispherical area around the area of interaction of a and a' and the other along the sides of the generally toric deformation of the initial plane of the interaction. For simplicity we will assume that the radii are of equal magnitude, though necessarily of different sense.

With an elastic collision, at some point all the momentum of A will be transferred to a, but in the case of a continuum a, at such point all the kinetic energy of A is transferred to the elastic potential energy of the stress and strain at the deformation of a. As the restorative forces in the plane of the interaction of $a\,a'$ exceed the compression force of a' on a, a force which is normal to the interaction plane, a will rebound and begin to work on a', which will travel in the opposite direction, eventually to be expelled from the plane of the initial impact.

We imagine this interaction now with another half continuum mirrored opposite *a*, so that the total system of the aa' interaction constitutes a resonant oscillation of a localized, ellipsoidal section of the combined continuum along the center axis aa', in which $m_a = m_{a'} = m_0$.

Using equation (1.6), we can state the linear inertial density, λ_0, of the continuum at the system as follows,

$$\lambda_0 = \frac{m_0}{r_0} = \frac{\text{ח}}{r_0^2} = \text{ח}\kappa_0^2 . \tag{1.31}$$

This indicates that the linear inertial density is equal to the inertial constant times the curvature of the deformation or strain, as shown in the last term. Assuming an isotropic Gaussian curvature, *k*, given by

$$k = \kappa_0^2 = \frac{1}{r_0^2}, \tag{1.32}$$

this means that mass is a measure of <u>linear</u> curvature, given by the angular wave number, κ, once again indirectly related to the length scale, as

$$m_0 = \frac{\text{ח}}{r_0} = \text{ח}\kappa_0 . \tag{1.33}$$

Now we stipulate that instead of a linear oscillation, we have a torsional oscillation of a small section of a generally rigid continuum about an axis, θ, so that at resonant frequency a wavelength of characteristic angular wave number, κ_0, develops. As a torque vector, θ flips in direction and oscillates in intensity with each change in the direction of the torsion. We next rotate the axis of torsion about a center of wavelength, such that the transverse momentum of θ carries the two nodes into a newly defined axis, ϕ, in the general helical path of the oscillation. Restorative forces prevent rotation of ϕ in θ beyond a point and the original oscillation of θ continues, now aligned as ϕ, between its original nodal poles and causing those displaced nodes to rotate about θ without entangling or twisting the continuum beyond a general range of one half π. As a result, the torque vector for ϕ does not oscillate as did θ. It rotates about θ, so that θ ceases to flip its direction and becomes a sustained angular momentum vector according to the rotation. Note that θ remains the primary torque, with ϕ as a derivative, so that the interaction between the two is non-commutative and there is a tendency over time for ϕ to realign as θ.

The motion can be crudely emulated by imagining a basketball held out in your two hands so that the label is facing up and away from you, i.e. it would be readable by someone facing you and looking down on it. The label represents the ultimate +θ direction and you can start the initial oscillation by moving your hands back and forth so as to rotate the ball about an axis through the center of the ball and label. When the ball is rotated so that your right hand is closest to your body, start the return counterclockwise cycle, but initiate a second rotation so that you start a new motion by 1) rotating the right hand over, left hand under and twisting 90 degrees ccw so that someone on your left can tilt their head to the right and read the label, 2) returning the hands to their first position, but twisting the ball back so that the label is facing you upside down, 3) rotating the left hand over, right hand under and twisting 90 degrees ccw again so that someone on your right can tilt their head to the left and read the label, and 4) returning the hands to their original position, but twisting the ball over so that the label is readable by someone at a distance in front of you, 5) continuing on to step number 1. Continue the process so that the motion between each step is smooth and the label remains ideally in the same plane or in a wide cone of even angle. Each hand will trace a figure 8 with each cycle, crossing at 45 degrees to the plane and in the direction of rotation with each half cycle. The label and its antipode represent the original boundaries of θ, now of ϕ, and rotate while avoiding entanglement. The a) left and right hands and the b) top and bottom of the ball when at step 2 oscillate between the top and bottom position, 90 degrees out of phase, while the motion of the hands between position 1 and 3 clearly shows, in the context of a continuum, the torsion involved in both a and b.

Such rotational oscillation mimics the rotation of our three dimensional clock developed in the previous section on kinematics. While locally constrained by the stress of an expanding spacetime, the wave phasing which manifests as spin is free to transform translationally and rotationally subject to field and particle interactions and perturbations.

The torsion forms a sinusoidal wave rotating about θ, with a sustained displacement of the initial θ nodes. In this context, the moments of maximum capacitive and inductive power of the wave generate a capacitive, C_ε, and an inductive, L_μ, torque (crossed from a position of zero displacement into the most immediate direction of increasing force, thus backwards in time for L_μ) that rotate with θ, 90 degrees out of phase with each other, C_ε generally parallel and L_μ generally anti-parallel. These torques interact with the nodes of ϕ to prevent realignment with θ. The rotation of the inductive torque generates an effective magnetic moment anti-parallel to the angular momentum vector of θ.

In the presence of an isotropic expansion, the characteristic inertial density and related mechanical impedance, given by

$$Z_0 = \lambda_0 c = \hbar \kappa_0 \omega_0 , \tag{1.34}$$

decrease over time, advancing the inductive moment and dropping the frequency. This results in the flip of the inductive torque to a generally parallel position behind the capacitive torque, which also flips the effective magnetic moment to parallel. A power transmission results anti-parallel to the inductive torque and the spin vector, along with a generation of charge, the latter of which is a measure of transverse wave momentum; we recognize this as beta decay. The decreased frequency is that of the proton, and that of the emitted wave is that of the electron. The rest frequency of the latter is a function of geometry and the expansion rate.

With a retarding of the capacitive moment, the capacitive torque flips anti-parallel, the oscillation becomes an anti-proton and emits a positron. The expansion of spacetime is more conducive to the inductive advance, resulting in a predominance of the proton-electron system.

Thus a geometrization of massive-particle mass involves the representation of quanta as three dimensional clocks and indicates that particle mass is a measure of the frequency of the clock. As the above continuum is a representation of 3-D space, its quantization represents an oscillation of a local section of space that is made discrete by the boundaries of its harmonic oscillation. If that oscillation is seen to be at resonant frequency, ω_0, then we have the following relationship to the wave speed in such continuum

$$\kappa_0 = \frac{\omega_0}{c} \tag{1.35}$$

which when substituted into equation (1.31) gives the following wave equation, where τ_0 is the tension force in the continuum,

$$\hbar\kappa_0^2 = \lambda_0 = \frac{1}{c^2}\tau_0 = \frac{\hbar\omega_0^2}{c^2} \tag{1.36}$$

where by canceling the inertial constants and substituting conventional derivatives for the Euler versions we have the dimensionally equivalent wave equation

$$\frac{\partial^2\psi}{\partial x_i^{\,2}} = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} \tag{1.37}$$

Finally, integrating equation (1.36) with respect to the wave number shows the basic wave nature of mass – energy equivalence as

$$-im_0 = \frac{\hbar\kappa_0^2}{i\kappa_0} = \frac{1}{c^2}\frac{\hbar\omega_0^2}{i\kappa_0} = -i\frac{E_0}{c^2} \tag{1.38}$$

From this discussion we can state some basic quantum dynamic properties of interest in terms of the inertial constant:

Interaction impulse = transverse wave momentum	$\Delta p = mc = \sqcap\omega$
Force – stress force = transverse wave force	$\tau = \sqcap\omega^2$
Action = spin angular momentum	$S = \hbar = \sqcap c = \sqcap\frac{\omega}{\kappa}$
Rest Mass	$m = \sqcap\frac{\omega}{c} = \sqcap\kappa$
Spin Energy	$E = mc^2 = \hbar\omega = \sqcap c\omega$

For the etymologically inclined, the word *mass* is from the German *massieren* meaning *to knead* dough, and evokes the notion of folding and stretching the dough with the heel of the hand, turning it 90^o, and repeating the process. This action forms gluten, allowing the dough to catch the gas of the leavening agent and expand. The symmetry of this scenario, with its orthogonal folding and rotation of dough, and the torsional rotation of the 3-D clock developed herein as an analogue of the fundamental rest mass rotational oscillation of spacetime is inescapable.

Gravitational Mass (Source)

In general relativity, gravitational source mass is converted from conventional units related to a force, M_{kg}, to units of length, r, as M_l, by the conversion factor of G_N/c^2, where G_N is Newton's gravitational constant and c^2 is the speed of light in a vacuum squared, both of which are taken as free parameters, as

$$r = M_l = \frac{G_N}{c^2} M_{kg} = \left(7.424x10^{-28}\ \tfrac{m}{kg}\right) M_{kg}\ . \tag{1.39}$$

here evaluated using the CODATA SI values.[4] This procedure facilitates computation, as when used in a metric, so that if M_l is the geometrized mass of an extreme Kerr black hole, the reduced circumference at the horizon is $r_h = M_l$.

It bears noting that the relationship between the two measures of mass is direct and appears to be classical, so that we can state a differential form

$$dr = dM_l = \frac{G_N}{c^2} dM_{kg} \tag{1.40}$$

We should acknowledge, however, the obvious and logical possibility that M_{kg} is an aggregation of some basic quantized units of mass of one or more magnitudes. Consideration of this equation using the smallest of rest-mass quanta, the electron, gives a linear measure of its mass in orders of magnitude of 10^{-58} meters. For the proton and neutron, the figure is slightly larger at 10^{-54} meters. However, all of these are much smaller than the Planck length of 10^{-35} the reputed smallest of determinable physical scales, raising possible theoretical questions about the use of equation (1.39) in determining a geometrized mass for individual quanta.

As previously discussed at equation (1.24), the mass of an individual quantum, a neutron, proton, electron, tau, or muon is related to its reduced Compton wavelength by the following, where the r_q is the reduced circumference and the norm of a polar coordinate system centered on q.

$$\lambda\!\!\!^{-}_{C,q} = \frac{\hbar}{c}\frac{1}{m_q} = \frac{ת}{m_q} = r_q\ . \tag{1.41}$$

In the SI system, ת (tav), evaluates as

$$\daleth \equiv \frac{\hbar}{c} = m_q r_q = 3.5176x10^{-43} kg \cdot m \tag{1.42}$$

In summary, in contrast to the direct relationship in the geometrization of mass in the classical application of general relativity, in quantum theory conventional mass is indirectly related to length. If we consider a relativistic quantum qualitatively, we know that the deBroglie wavelength decreases as the relativistic mass and the particles momentum increases, indicating once again the inverse relationship of mass and length.

3 – Derivation of Newton's Gravitational Law with Quantum and General Relativistic Principles

We would like to derive Newton's Gravitational Law from quantum principles, while in keeping with the principles of general relativity. The quantum principle we are interested in is that of fundamental discrete units or quantities of rest mass. This means that we seek to express the gravitational force, F, of his law as a product of 1) the number, n_a, of some as yet unknown fundamental discrete units of mass, m_0, in two aggregate bodies of mass, M_a, 2) the curvature of space, k, expressed as the inverse square of the massive bodies separation in numbers, n_r, of some as yet unknown minimum unit of length specifically of a reduced circumference, r_0, and 3) a fundamental discrete unit or quantum differential of gravitational force, dG_0, as

$$F_{m_1 m_2 k} = n_{M1} n_{M2} n_r^{-2} dG_0 \tag{2.1}$$

We state Newton's Law, where G_N is Newton's gravitational constant, conventionally considered a free parameter, as

$$F_{M_1 M_2 k} = \frac{M_1 M_2}{R^2} G_N = M_1 M_2 k G_N \, . \tag{2.2}$$

Assuming a 3-space that is isotropic with respect to a source mass, M_1, here we have made use of the observation that the inverse square component of the distance of separation, R, of M_1 and M_2 is the reduced circumference of the spacetime around M_1, making the inverse of the square of R the measure of the Gaussian curvature, k, at the location of M_2, using

$$k = \frac{1}{R^2} \, . \tag{2.3}$$

The left hand side of equation (2.2) represents a force. Some reflection will tell us that if it is to be related to general relativity, the right hand side must represent the product of a 4-stress, T, and an area, A, or in keeping with the last paragraph, an inverse curvature. Thus this equation is dimensionally equivalent to

$$F = T\mathrm{A} = Tk^{-1} \, . \tag{2.4}$$

Some rearrangement gives us a scalar form

$$k = F^{-1} T \tag{2.5}$$

where the curvature of spacetime given by the left term is related to the stress-energy density of the right by the inverse force. This is thus related to the field equation of general relativity, customarily expressed in tensor form as

$$G_{\alpha\beta} = -8\pi G_N T_{\alpha\beta} \tag{2.6}$$

where the Einstein curvature tensor on the left is similarly related to the stress-energy tensor on the right by the geometrically based numerical coefficient and Newton's gravitational constant, which we will see contains a force differential.

Analyzing G_N dimensionally, we know it has the dimensions of distance, r, cubed divided by the product of a mass, m, and time, t, squared. If it in fact conceals a force differential, extracting that force in the third term shows G_N to be the product of that force and the inverse square of a linear inertial density, λ, as

$$G_N = \frac{r^3}{mt^2} = \frac{r^2}{m^2}\frac{mr}{t^2} = \lambda^{-2} dF \tag{2.7}$$

The inverse inertial densities in Newton's constant then convert the product of the masses on the right side of Einstein's field equation (2.6), of which there are two, one in the force differential and one in the stress tensor, to the product of two distances. The result, however, does not give the dimensions of curvature. With respect to equation (2.5), which has an inverse force on the right, the G_N as shown in equation (2.7) has a direct force. Using the fundamental identity of space and time as shown in equation (0.7), we can make the following dimensional substitutions into equation (2.7),

$$G_N = \frac{r^3}{mt^2} = \frac{(it)^3}{m(-ir)t} = \frac{t^2}{mr} = dF^{-1} \tag{2.8}$$

which converts G_N to an inverse force and equation (2.6) assumes the dimensional form of equation (2.5).

Expressed as a force, gravity is centrally directed toward the bodies of mass and within the context of a flat spacetime, assumed to be isotropic about each. The curvature in such conditions is considered generally spherical, so that some rearrangement of equation (2.4) in the absence of any rotation of the two bodies about each other, results in a centripetal gravitational tension stress, f_3, where the subscript indicates the dimensional order of the stress

$$f_3 = \frac{F}{\mathrm{A}} = Fk\,. \tag{2.9}$$

The stress in the case of general relativity is a 4-stress, however, so that we are looking for a formulation that makes explicit the relationship between a 3-stress and a 4-stress, T_4

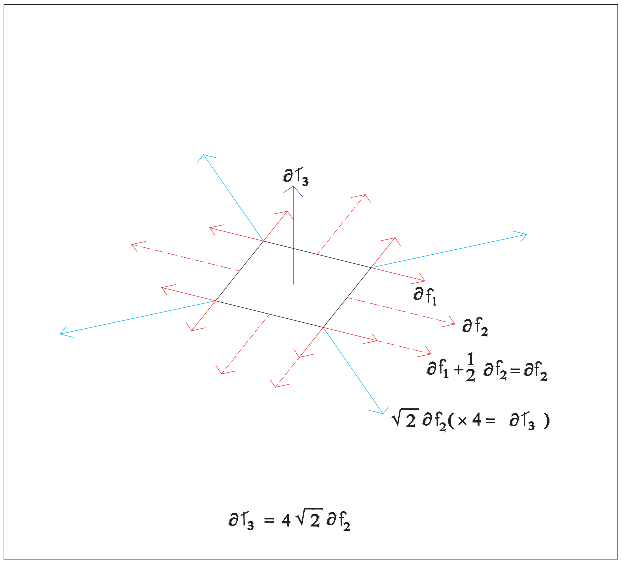
$$f_3 \equiv \frac{F}{A} \sim T \equiv T_4\,. \tag{2.10}$$

Additionally, we are interested in the 4-stress associated with an accelerating expansion of space, so we take a closer look at the geometry of stress, specifically of isotropic expansion stress. We examine the case of energy density - stress in an n-manifold that is expanding in response to its expanding n+1-core. First, in Stress Diagram 1 we examine the differential area of a 2-sphere on a 3-ball, such as an expanding balloon. We imagine that the balloon is expanding due to a differential pressure normal to the balloon surface, so that there is a 3-stress (tension), dT_3, orthogonal to the balloon's surface, the 2-sphere. We look at a differential square on the surface of the balloon and see that the sum of the

2-stress (transverse or shear), df_2, in the balloon surface at that locus should be equal to the orthogonal tension stress, or

$$dT_3 = \gamma_2 df_2 \tag{2.11}$$

where γ_2 is a geometric factor summing the shear stress.



Stress Diagram 1
Figure 12

It is the displacement of the vertices of the square that defines the change, so instead of a normal unit vector to each mid-edge of the differential square, we stipulate a ½ vector at each vertex, along with a ½ extension or shear vector from the adjacent edge, giving a total of 8, 1 vectors at the four vertices. With a total of 4 resultants of the vector pairs at each vertex, we have

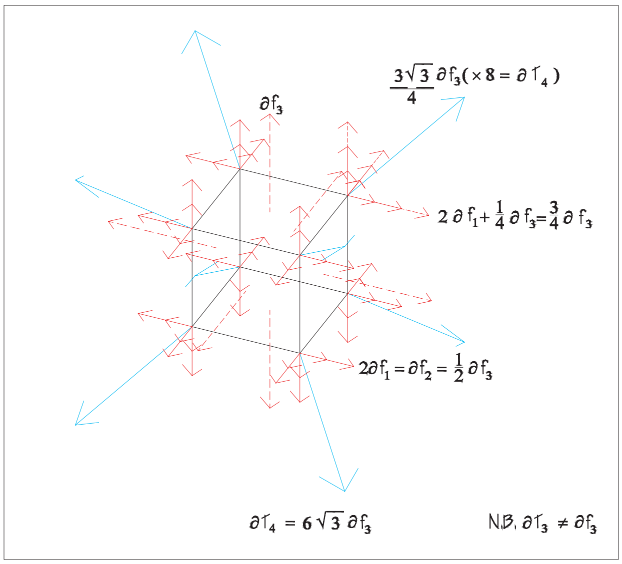
$$\gamma_2 = 4\left(\sqrt{1^2 + 1^2}\right) = 4\sqrt{2} \tag{2.12}$$

and equation (2.11) becomes

$$dT_3 = 4\sqrt{2}df_2 \tag{2.13}$$

Extending this approach with analogous elasticity conditions to a 3-space on a 4-core, in Stress Diagram 2 we have a 4-stress, (which necessarily cannot be shown) normal to and equal to an isotropic 3-stress, as

$$dT_4 = \gamma_3 df_3 . \tag{2.14}$$



Stress Diagram 2
Figure 13

This time we consider a differential cube, and instead of the customary assignment of an orthonormal tension stress vector to the center of each of the faces of the cube, we assign a quarter of each normal vector to the corners of each face, collinear with and equal to the shear vectors of the two adjacent surfaces. This is equivalent to a Poisson’s ratio of 1/3. The sum of these ¼ tension vectors and the two parallel ¼ shear vectors is a ¾ vector, so that there are 3, ¾ orthogonal stress vectors at each vertex. The resultant of the three orthogonal components at each corner then, aligned with the cubic diagonal, is the total stress contributed to each of the 8 vertices by an isotropic stress, so that the geometric factor relating the stresses in equation (2.14) is

$$\gamma_3 = 8\left(\sqrt{\left(\tfrac{3}{4}\right)^2 + \left(\tfrac{3}{4}\right)^2 + \left(\tfrac{3}{4}\right)^2}\right) = 8\frac{3\sqrt{3}}{4} = 6\sqrt{3} \tag{2.15}$$

and equation (2.14) becomes

$$dT_4 = 6\sqrt{3}df_3 \tag{2.16}$$

Next we examine a scalar expression of the equation (2.10) in light of this adjustment, where we specify that A_0 is a fundamental quantum unit area,

$$\gamma_3^{-1}T = \frac{F}{\mathrm{A}_0}, \tag{2.17}$$

with the derivatives for an invariant T being

$$\gamma_3^{-1}dT = \frac{\partial T}{\partial F}dF - \frac{\partial T}{\partial \mathrm{A}}d\mathrm{A} = \frac{1}{\mathrm{A}_0}dF - \frac{F}{\mathrm{A}_0^2}d\mathrm{A} = 0\,. \tag{2.18}$$

Separating and inverting this function we have the two following differential equations, the first of which is straight forward,

$$dF = \gamma_3^{-1}\mathrm{A}_0 dT \equiv \left(\gamma_3^{-1}\kappa_0^{-2}dT\right) \tag{2.19}$$

and the second one expressing various parsings of interest, especially those in which the stress force is removed from the equation,

$$\begin{aligned} d\mathrm{A} &= -\gamma_3^{-1}\frac{\mathrm{A}_0^2}{F}dT = -\frac{F}{\gamma_3^{-1}T^2}dT = -\frac{\mathrm{A}_0}{T}dT \\ &= -\mathrm{A}_0 d\ln T \equiv \left(-\kappa_0^{-2}d\ln T\right) \end{aligned} \tag{2.20}$$

According to the above specifications a quantum formulation for Newton’s Law, as previously stated, would be

$$F_{m_1 m_2 k} = n_{M1} n_{M2} n_r^{-2} dG_0 \,. \tag{2.21}$$

An aggregate mass is the product of the number of quanta in that aggregate times the fundamental unit of mass or with rearrangement

$$n_{Ma} = \frac{M_a}{m_0} \tag{2.22}$$

and the reduced circumference of the separation of the two bodies of mass is the product of the number of unit lengths in that separation and the minimum or quantum unit length, or

$$n_r = \frac{R}{r_0} \,. \tag{2.23}$$

Substituting equation (2.22) and equation (2.23) into equation (2.21), noting the dimensional equivalence of the bracketed term to equation (2.7), gives

$$F_{M_1 M_2 k} = \frac{M_1 M_2}{R^2} \left(\frac{r_0^2}{m_0^2} dG_0 \right) \tag{2.24}$$

Assuming that the gravitational quantum is equivalent to the formulation from equation (2.19) and substituting from its middle term, gives the following, in which the stress differential is normalized in its relationship to dG_0 as $dT_0 = 1$,

$$F_{M_1 M_2 k} = \frac{M_1 M_2}{R^2} \left(\gamma_3^{-1} \frac{r_0^4}{m_0^2} dT_0 \right) = \frac{M_1 M_2}{R^2} G_N \,. \tag{2.25}$$

In keeping with earlier development, we restate the relationship between the above postulated quantum mass, m_0, and length, r_0, the latter stated as the reduced Compton wavelength,

$$m_0 = \frac{\hbar}{c}\bar{\lambda}_{C,0}^{-1} = \frac{\hbar}{c}r_0^{-1} = \frac{\text{ח}}{r_0}. \tag{2.26}$$

We substitute from equation (2.26) into the bracketed term of equations (2.24) and (2.25), and get

$$G_N = \frac{dG_0}{\lambda_0^2} = \frac{r_0^4}{\text{ח}^2}dG_0 = \gamma_3^{-1}\frac{r_0^6}{\text{ח}^2}dT_0, \tag{2.27}$$

which in a natural system simply equals γ_3^{-1}.

After some rearrangement, we have

$$r_0 = \left(\gamma_3 \text{ח}^2 G_N dT_0^{-1}\right)^{1/6}. \tag{2.28}$$

Since with respect to dG_0, dT_0 equals 1, and as we know the other invariants in the right hand term, we can solve for r_0, and find that in the SI system it equals the reduced Compton wavelength of the neutron or

$$r_{0n} = 2.100246...x10^{-16}m \cong 2.10019...x10^{-16}m = \bar{\lambda}_{C,n} \tag{2.29}$$

within the standard uncertainty for G_N. The “*n*” in the subscript “0*n*” is redundant and simply emphasizes the neutron scale as the fundamental, quantum scale. All other values for the fundamental properties incorporate and can be computed from this value. Therefore, the fundamental gravitational mass is the neutron mass or

$$m_0 = m_n = 1.67492...x10^{-27}kg. \tag{2.30}$$

This is not stating that the neutron is the only particle responsible for the generation of quantum gravity. In the full development of this model, the proton is seen to be a neutron which has undergone a frequency drop due to the transmission of a portion of its power as the electron in the process of beta decay. It is the derivative of the wave force of the fundamental oscillation with respect to stress that is the gravitational quantum.

The gravitational quantum then is variously

$$dG_0 = \gamma_3^{-1}\kappa_{0n}^{-2} dT_0 = \gamma_3^{-1} \mathrm{A}_{0n} dT_0 = \gamma_3^{-1} T_0 d\mathrm{A} = 4.244...x10^{-33}\, N \,, \tag{2.31}$$

where the last algebraic term makes use of equation (2.20).

Some rearrangement gives

$$T_0 = \gamma_3 \frac{dG_0}{d\mathrm{A}} = \frac{\mathrm{A}_{0n}}{d\mathrm{A}} dT_0 \tag{2.32}$$

With this development, we can get the spin energy density-stress, T_0, of the neutron, which we assume to be a quantum waveform, where E_{0n} is the spin energy of the neutron and

$$\tau_0 = \hbar\omega_0^2 \tag{2.33}$$

is the transverse wave force of the oscillation,

$$\gamma_3^{-1} T_0 = \frac{E_{0n}}{r_{0n}^3} = \frac{m_n c^2}{r_{0n}^3} = \frac{\hbar\omega_0^2}{r_{0n}^2} = \frac{\tau_0}{r_{0n}^2} = 1.625...x10^{37}\, N/m^2 \,. \tag{2.34}$$

Substituting this into equation (2.31) for the gravitational quantum and rearranging, we get the following expression and value for the differential of the unit area, $d\mathrm{A}_0 = d\mathrm{A}$, which we find is equal to the Planck area,

$$d\mathrm{A}_0 = \gamma_3 T_0^{-1} dG_0 = \mathrm{A}_{Pl} = 2.6116...x10^{-70} m^2 \tag{2.35}$$

This analysis indicates that Newton's gravitational constant contains a quantum differential, and that the neutron scale is the fundamental scale of an expanded spacetime. It also indicates a relationship to the Planck scale, and we would like to determine more of that relationship next.

4 – Analysis of the Relationship between the Neutron and the Planck Scale

If we use the conventional geometrization factor from general relativity for mass, G_N/c^2, for the neutron we get a length measure of a hypothetical quantum black hole horizon as

$$r_{hn} = m_{l,n} = \frac{G_N}{c^2} m_n = 1.243...x10^{-54} m\ . \tag{3.1}$$

Comparing this with the neutron reduced Compton, we get the dimensionless number

$$\frac{m_{l,n}}{\lambdabar_{C,n}} = \frac{r_{hn}}{\lambdabar_{C,n}} = 5.92...x10^{-39}\ . \tag{3.2}$$

Squaring equation (3.1) to get the inverse curvature of a hypothetical quantum inertial sink at that scale gives

$$r_{hn}^2 = 1.545...x10^{-108} \tag{3.3}$$

which is related to the Planck area by the same ratio or

$$\frac{r_{hn}^2}{A_{Pl}} = 5.92...x10^{-39}\,. \tag{3.4}$$

It bears noting that this is in the range of the factor separating the gravitational and the strong interactions.

Using the structure for Newton's constant developed above, we analyze the conventional geometrization factor, where we make use of the classical wave relationship,

$$\tau_0 = \lambda_0 c^2 \tag{3.5}$$

in which τ_0 is the linear tension force and in this case the transverse wave force in a wave bearing medium, λ_0 is the linear inertial density of that medium and c is its speed of wave propagation. We find that the conventional conversion factor is equal to the differential of the natural log of the expansion stress divided by the linear inertial density,

$$\frac{G_N}{c^2} = \left(\frac{dG_0}{\lambda_0^2}\right)\frac{1}{c^2} = \frac{1}{\lambda_0}\left(\gamma_3^{-1}\frac{r_{0n}^2}{\lambda_0 c^2}\right)dT_0 = \frac{1}{\lambda_0}\left(\gamma_3^{-1}\frac{r_{0n}^2}{\tau_0}\right)dT_0 = \frac{1}{\lambda_0 T_0}dT_0 \tag{3.6}$$

$$\frac{G_N}{c^2} = \frac{1}{\lambda_0 T_0}dT_0 = \frac{d\ln T_0}{\lambda_0} = \frac{\lambda\!\!\!^{-}_{C,n}}{m_n}d\ln T_0 \tag{3.7}$$

Using CODATA values for the neutron mass and reduced Compton to determine λ_0, we can solve for $d\ln T_0$ and get the factor found in equations (3.2) and (3.4)

$$d\ln T_0 = T_0^{-1}dT_0 = \gamma_3^{-1}\frac{r_{0n}^3}{m_n c^2}dT_0 = 5.92146...x10^{-39} \tag{3.8}$$

Inverting and multiplying through by $dT_0 = 1$ gives the value of T_0,

$$T_0 = \gamma_3\frac{m_n c^2}{A_{0n} r_{0n}} = \gamma_3\frac{\lambda_0 c^2}{A_{0n}} = 1.6888...x10^{38}\,N/m^2 \tag{3.9}$$

from which we can get the transverse quantum wave force of the neutron

$$\tau_{0n} = \gamma_3^{-1} T_0 \mathrm{A}_{0n} = 7.1676...x10^5\, N \tag{3.10}$$

With the gravitational quantum as the differential of the quantum transverse wave force with respect to differential stress, $\tau'(T)$, we have the ratio of that differential and the wave force itself, $\tau(T)$, or equation (2.31) over equation (3.10)

$$d\ln T_0 = \frac{\tau'(T)}{\tau(T)} = \frac{d\tau_0}{\tau_{0n}} = \frac{dG_0}{\tau_{0n}} = 5.92146...x10^{-39} \tag{3.11}$$

which is the ratio of the gravitational and the strong interactions.

Rearranging equation (3.10) and taking the derivative of inverse curvature with respect to the isotropic stress results in an evaluation equal to the Planck area,

$$d\mathrm{A}_0 = -\gamma_3 \frac{\tau_{0n}}{T_0^2} dT_0 = -\mathrm{A}_{0n} d\ln T_0 = -\mathrm{A}_{Pl}\,. \tag{3.12}$$

once again indicating that the Planck area represents a differential of expansion stress. To verify this statement, we substitute equations (2.33), (3.5), and (3.9) for the expansion force and stress into the second term here and find

$$\begin{aligned} d\mathrm{A}_0 &= -\gamma_3 \frac{\sqcap\omega_{0n}^2}{\gamma_3^2 \lambda_0{}^2 c^4 \mathrm{A}_0^{-2}} dT_0 = \left(-\gamma_3^{-1} \frac{\mathrm{A}_0}{\lambda_0{}^2} dT_0 \right) \frac{\sqcap\omega_{0n}^2 r_{0n}^2}{c^4} \\ &= -G_N \frac{\sqcap c^2}{c^4} = -G_N \frac{\hbar}{c^3} \end{aligned} \tag{3.13}$$

From this analysis of the differential nature of the Planck area and the endnote comments,[i] which suggest expansion along a hyperbolic manifold, from equation (3.12) we can show the Planck length as a differential value, as

$$dr_0 = \left| d\mathrm{A}_0 \right|^{\frac{1}{2}} = r_{0n}\sqrt{d\ln T_0} = r_{Pl} = 1.6161...x10^{-35}m\,. \qquad (3.14)$$

5 – Cosmological Implications

Basic to our discussion is the assumption that spacetime is expanding relative to our local frame of reference. This means that over time a local fixed unit length standard becomes an ever decreasing proportion of some linear measure of the cosmic extent. If we project backwards in time, we can assume that at some point that measure of cosmic extent was equal to the current local length standard or unity.

The current concept of a big bang start of cosmic spacetime expansion implies an initial condition of maximum inertial density, possibly infinite, which decreases with the expansion of space from an extremely small volume, possibly zero, i.e. from a singularity. Instead of emergence from a singularity, the space component of spacetime can be modeled as a boundary on the next higher dimensional manifold itself under expansion, analogous to a circle drawn on the surface of an expanding balloon. Alternately, we might imagine a spherical balloon of fixed size with a circular wave emanating from one spot, widening in diameter as it approaches an equator before shrinking again as it nears the antipode. An analogous inertial spacetime oscillates on a cosmic scale between a maximum density and rarification, between a maximum compression and maximum extension. The fact that the expansion appears to be accelerating indicates that the expansion rate is best understood exponentially. We can then take the condition of maximum density as unity instead of as a singularity, and gauge any expansion with respect to that unity for A_0 and r_0 as inversely related to the associated increase in stress T_0 due to expansion according to equations (3.12) and (3.14).

The current expansion factor, $\kappa_{\exp}$, the ratio of the current fundamental neutron scale to the Planck length, is equal to the inverse square root of the differential natural log of the expansion stress,

$$\kappa_{\exp} = \frac{r_{0n}}{dr_0} = \sqrt{d \ln T_0}^{-1} = 1.29952...x10^{19} \tag{4.1}$$

As this expansion is at an exponential rate, in terms of doubling from an initial condition of maximum density equal to the linear inertial density of the neutron scale, λ_0, with time and space normalized, in terms of the whole or an arbitrary unit standard, cosmic expansion, C_x, is

$$C_x = \ln 2\left(\kappa_{\exp}\right) = 9.00764...x10^{18} \text{ light seconds } = 2.8544...x10^{11} \text{ light years} \tag{4.2}$$

Note that the last term would indicate, if interpreted as a straight line increase at the speed of light, an expansion age of the cosmos of 285.44 billion years.

An exponential expansion rate, X_e, derived in the full development of this model and shown to equate to a predicted Hubble rate of 72.791 km/mps/s and supported by independent studies as 73 km/mps/s +/- 8 km [7] and 72 km/mps/s [8], shows the change in unit scale per second as

$$X_e = H_0 = \frac{\Delta r_0}{r_0} \Big/ \text{second} = 2.35896...x10^{-18} / \text{ s} \tag{4.3}$$

If we interpret this as a straight line expansion rate from an initial singularity, inverting would give the age of the cosmos in current units as

$$X_e^{-1} = 13.433 \text{ billion years} \tag{4.4}$$

However, if the Hubble rate is exponential or compounding, the following gives the Hubble time, τ_H, as a time in current units for a doubling in spatial linear extent, or

$$\tau_H = \ln 2 X_e^{-1} = 9.311 \text{ billion years} \tag{4.5}$$

The product of the expansion rate and the expansion factor is the number of doublings or

$$X_e \kappa_{\exp} = 30.655... \text{ doublings } = 285.43 \text{ billion years} \tag{4.6}$$

Following this logic, if the wavelength of the cosmic microwave background is approximately 3.3*mm* and indicates an expansion along with spacetime from a primal epoch of beta decay as gauged by the electron Compton wavelength, $\lambda_{C,e}$, dividing the natural log of such expansion by the natural log of 2 gives the number of doublings based on those parameters or

$$\ln\left(\frac{.0033}{\lambda_{C,e}}\right)(\ln 2)^{-1} = \frac{\ln 1.360...x10^9}{\ln 2} = 30.34... \text{ doublings } = 282.5 \text{ billion years} \tag{4.7}$$

in very close agreement with equation (4.6).

This observation indicates that r_0, related to the reduced electron Compton wavelength, $\bar{\lambda}_{C,e}$, by the ratio of the neutron to electron Compton wavelength of 0.000543…, remains stable as spacetime and the CMB expands and indicates that such quanta did not have a geometry of the Planck scale at an early epoch, which instead of starting from a singularity with all the physical dilemma that entails, started expansion from a maximum finite density. The Planck length, then, is the ratio of the neutron reduced Compton and the cosmic extension from an initial compact condition of maximum density, and continues to decrease with expansion.

Alternately, but not contradictory, if we think of the cosmic extent of 3-space as a fixed unit, what appears mathematically from a local perspective as expansion is from the universal perspective a process of regional and local concentration of inertial density. With respect to our analogy of the fixed balloon above, the linear (and area) density of the balloon in the absence of a wave is invariant over the surface of the sphere, but a wave moving over its surface creates a density differential at the wave front, increasing as it approaches a pole and antipole and decreasing as it approaches an equator. From the reference frame of the traveling wave front approaching the poles, the stress related to the wave front,T_0, increases and r_0, as a related unit standard which in the case of the balloon we might give as the distance perpendicular to the given polar diameter, decreases over time. The ratio of r_0 with respect to the balloon's extent, B_x, its radius at the equator, represents a decreasing differential length, dr_0, and can be expressed as the cosine of the angle of declination of the wave front.

The wave front in this analogy represents the current local quantum scale given by r_{0n}. If we were to rotate the balloon about the given polar axis at the same frequency as the wave's movement over its face, each point in the wave front would mimic the action of our 3-D clock. From either of the above perspectives, the energy per cosmic extent is invariant and cosmological red-shift is apparent, and in neither case is the Planck scale a fixed discrete scale.

Black Hole Metrics

Assuming that the above and supportive analysis does indicate that the neutron is a quantum inertial sink, but not a quantum black “hole”, then a maximum linear inertial density is given by

$$\lambda_0 = \frac{m_n}{r_{0n}} = 7.975...x10^{-12}\, kg\,/\,m \tag{4.8}$$

This would seem quite small, but for its bulk implications. For a volume density, we would figure the number of hypothetical fundamental rest mass quanta per volume of such quanta, tightly packed. Using a packing system of one sphere with twelve contacting identical spheres, and disregarding any expansive effects of spin, charge, etc., we can compute the theoretical maximum density and find that it equals roughly

$$2.2549...x10^{46}\, quanta\ /\ m^3 \tag{4.9}$$

Inverting the neutron mass gives the number of such quanta per kilogram or

$$5.9704...x10^{26}\, quanta\ /\ kg \tag{4.10}$$

for a maximum theoretical density of

$$3.7768...x10^{19}\, kg\ /\ m^3 \tag{4.11}$$

or a density per sphere of one meter radius

$$\rho_{sphere} = 1.5820...x10^{20}\,kg\ /\ meter\ sphere \qquad (4.12)$$

From this we can find a threshold black hole mass, $M_{kg,TBH}$ for an aggregation of quanta by using the following for a flat Euclidean space, where r_{Max} is the reduced circumference of a celestial body of maximum density,

$$r_{Max} = \left(\frac{M_{kg,TBH}}{\rho_{sphere}}\right)^{\frac{1}{3}} \qquad (4.13)$$

Assuming $r_h = M_l$ as with an extreme Kerr spacetime

$$\frac{G_N}{c^2} M_{kg,TBH} = M_l = r_h = r_{Max} = \left(\frac{M_{kg,TBH}}{\rho_{sphere}}\right)^{\frac{1}{3}} \qquad (4.14)$$

for an extreme Kerr horizon gives

$$M_{kg,TBH} = \left(\frac{c^6}{G_N{}^3 \rho_{sphere}}\right)^{\frac{1}{2}} = 3.930x10^{30}\,kg \qquad (4.15)$$

which using the above density gives us the evaluations in the following table or approximately two solar masses for the threshold.

Here in column 3, from equation (4.13) we compute the r_{Max} for various celestial bodies, Earth, Sun, Milky Way galactic BH and Virgo cluster BH, and include the theoretical threshold size black hole and the Universe, as listed in column 1. “Flat Spacetime” does not specify that the pertinent body has no curvature effect on the surrounding spacetime, but rather that the curvature of individual quanta, i.e. quantum gravity, is not effected by the aggregate mass and remains the same as for an individual quantum in isolation in flat spacetime, i.e. there is no assumed collapse of each quantum waveform toward a quantum singularity, though there may be a state similar to a Bose-Einstein condensate. The fourth column gives the reduced circumference at the horizon of an extreme, charge free, Kerr black hole according to the conventional interpretation of general relativity. The fifth column simply makes explicit whether the third column figure resides within the fourth. This indicates that the rest mass quanta inside a black hole horizon could congregate at maximum density without precipitating a singularity.

	Mass in kg	Radius, r_{Max} in m, Density = ρ_{Sphere} Flat Spacetime	Mass in meters $M_l = \frac{G_N}{c^2} M_{kg} = r_h$	Is r within M_l = r_h at Horizon?
Earth	$*5.9742x10^{24}$	33.55	$4.44x10^{-3}$	No
Sun	$*1.989x10^{30}$	$2.325x10^{3}$	$1.477x10^{3}$	No
Kerr BH threshold	$3.930x10^{30}$	$2.913x10^{3}$	$2.913x10^{3}$	At Horizon
Milky Way	$*5.2x10^{36}$	$3.20x10^{5}$	$3.86x10^{9}$	Yes
Virgo cluster	$*6x10^{39}$	$3.36x10^{6}$	$4.45x10^{12}$	Yes
Universe	$1.67x10^{53} = 10^{80}\,nucleon$	$1.02x10^{11}$	$1.24x10^{26} = 13.1x10^{9}$ light yrs	Yes

*[5] Figures from Exploring Black Holes, by Taylor and Wheeler, Addison Wesley Longman, 2000

Table 2 - Chart of Various Celestial Mass Geometrizations

Of interest is the fact that the universe appears to be within its own horizon, which conventionally would tend to imply that its constituents should be contracting, and that there are black holes within black holes. Also the mass in meters being equal to the reputed age of the universe times the speed of light seems a bit serendipitous unless of course that mass, i.e. the number of currently theorized nucleons, was estimated using the above geometrization equation. But this figure is not the currently estimated extent of the universe, which has a lower end range of 78 billion light years or 24 Gpc according to a study by Cornish, et al.[6] Finally it is noted that the hypothetical mass of the known universe at maximum density and a radius of 102 million kilometers, would fit inside the earth's solar orbit in flat spacetime.

6 – The Quantum Metric

We turn now to the metric, specifically a chargeless extreme Kerr metric in the equatorial plane (the ϕ coordinates are suppressed), in which the angular momentum parameter, *a*, is equal to the horizon reduced circumference and the geometrized mass, or $a = r_h = M_l$.

The timelike metric at the horizon is

$$d\tau^2 = \left(1 - \frac{2M_l}{r_h}\right)dt^2 + \frac{4M_l a}{r_h}dtd\theta - \frac{dr^2}{\left(1 - \frac{2M_l}{r_h} + \frac{a^2}{r_h{}^2}\right)} - \left(1 + \frac{a^2}{r_h{}^2} + \frac{2M_l a^2}{r_h{}^3}\right)r_h{}^2 d\theta^2 \quad (5.1)$$

Substituting for $a = M_l$ gives

$$d\tau^2 = \left(1 - \frac{2M_l}{r_h}\right)dt^2 + \frac{4M_l^2}{r_h}dtd\theta - \frac{dr^2}{\left(1 - \frac{M_l}{r_h}\right)^2} - \left(r_h{}^2 + M_l^2 + \frac{2M_l^3}{r_h}\right)d\theta^2 \quad (5.2)$$

We make the following observation concerning the dr^2 term. While the conventional interpretation is that the term goes to infinity as the denominator approaches zero, and any infalling test particle transits the horizon, the math can also be interpreted in terms of a limit for radial motion. A mathematical conflation is at work in the formulation, since the differentials are deemed to approach zero in the limit, but are effectively treated as dimensional units, i.e. equal to one of some infinitesimal scale. This is necessary since the product of a non zero co-efficient and a zero differential at the limit would be zero. This is warranted since we find a similar non-zero differential without a coefficient on the left side of the equation.

This is contradicted, however, if the metric component represented by the differential has a natural limit where it is necessarily zero. Thus if the horizon in an extreme Kerr spacetime represents that limit, *dr* equals zero at the limit of that horizon coincident with the term in the denominator, the coefficient and the differential cancel. The result is simply -1 as shown below, which when factored gives an imaginary or orthogonal sense, i.e. it rotates any differential change into tangency. The horizon, then, is effectively a physical asymptote. Thus at the event horizon, where $r = r_h = M_l$ this simplifies to

$$d\tau^2 = -dt^2 + 4r_h dt d\theta - (2r)^2 d\theta^2 - dr^2 = (idt - i2r_h d\theta)^2 + (idr)^2 \tag{5.3}$$

This can be factored as a complex number and its conjugate

$$d\tau^2 = \left[(idt - i2r_h d\theta) + i(idr)\right]\left[(idt - i2r_h d\theta) - i(idr)\right] \tag{5.4}$$

or can be simplified as follows,

$$d\tau^2 = \left[(idt - i2r_h d\theta) - dr_h\right]\left[(idt - i2r_h d\theta) + dr_h\right] \tag{5.5}$$

where r_h is the reduced circumference at the horizon and $dr_h = 0$ is a zero vector with respect to the radial, giving a proper time of

$$d\tau = \pm i\left(dt - 2r_h d\theta\right) \tag{5.6}$$

If we assume that for bookkeeper time the differential is in the plane of the horizon, and time as developed earlier flows with the rotational motion of the ergosphere, so that

$$dt = r_h d\theta \tag{5.7}$$

then the proper time is found to flow orthogonally to that rotational motion, into the negative and positive ϕ coordinates, since

$$d\tau = \mp i dt \tag{5.8}$$

This will be significant in our statement of the quantum metric.

From this perspective, at the static limit and the start of the ergosphere, where $r = 2M_l$, pure radial motion is no longer possible, and a rotational component or frame dragging element is injected into the equation so that at the event horizon, all motion is rotational as indicated by the "imaginary" or orthogonal senses. Note that if we consider spacetime as an inertio-elastic continuum, frame dragging is simply the wave strain associated with a rotational waveform, be it macrocosmic or quantum. Instead of gravitational collapse, this argues that any incremental matter or light accruing to the inertial sink is smeared out and bound at the horizon in a state resembling a Bose-Einstein condensate.

We now get to the meat of the matter with an expression of the quantum metric. The dynamics of the quantum waveform is not extremely complicated, but it does involve some rather lengthy, non-standard analysis using methods of complex classical wave physics extended to 4 dimensions, and is beyond the scope of the present discussion. We will simply state that its kinematics prevent the orientation entanglement condition.

With reference to Quantum Inertial Sink Diagram 1, the timelike quantum metric is given as a modified chargeless extreme Kerr metric. The modification is in the ϕ coordinates as shown here, where the quantum mass has been explicitly geometrized as r_{0n},

$$d\tau^2 = \left(1 - \frac{2r_{0n}}{r_{0n}}\right)dt^2 + \frac{4{r_{0n}}^2}{r_{0n}}dtd\theta - \frac{dr^2}{\left(1 - \frac{r_{0n}}{r_{0n}}\right)^2} - R^2 d\theta^2 \mp \left\{\left(e^{\pm i(\omega_0 t \mp \theta)} L d\phi\right)^2\right\} \tag{5.9}$$

The caveat stated above concerning the limit of radial motion represented by r_{0n} remains. In the last term, the complex exponential is defined as

$$\begin{aligned} e^{\pm i(\omega_0 t \mp \theta)} &= \mathrm{Re}\left(e^{\pm i(\omega_0 t \mp \theta)}\right) = \cos_{ccw}\left(\omega_0 t + \theta\right) \text{ or } \cos_{cw}\left(\omega_0 t + \theta\right) \\ &= \cos\left(\omega_0\left(+t\right) - \theta\right) \text{or } \cos\left(\omega_0\left(-t\right) + \theta\right) \end{aligned} \tag{5.10}$$

Either the real or the imaginary part could of course be used. The *ccw* term indicates rotation in the upper hemisphere according to the right hand rule, while the *cw* term indicates clockwise rotation in the bottom hemisphere according to the left hand rule, when viewed from the exterior of the corresponding rotational pole.

The plus and minus curly bracket has the following definition and indicates a flipping of the sign of the $d\phi$ vector, with every π rotation of θ, plus being parallel and minus being anti-parallel with respect to the RHR spin axial vector. It thus performs a function similar to a mathematical spin matrix.

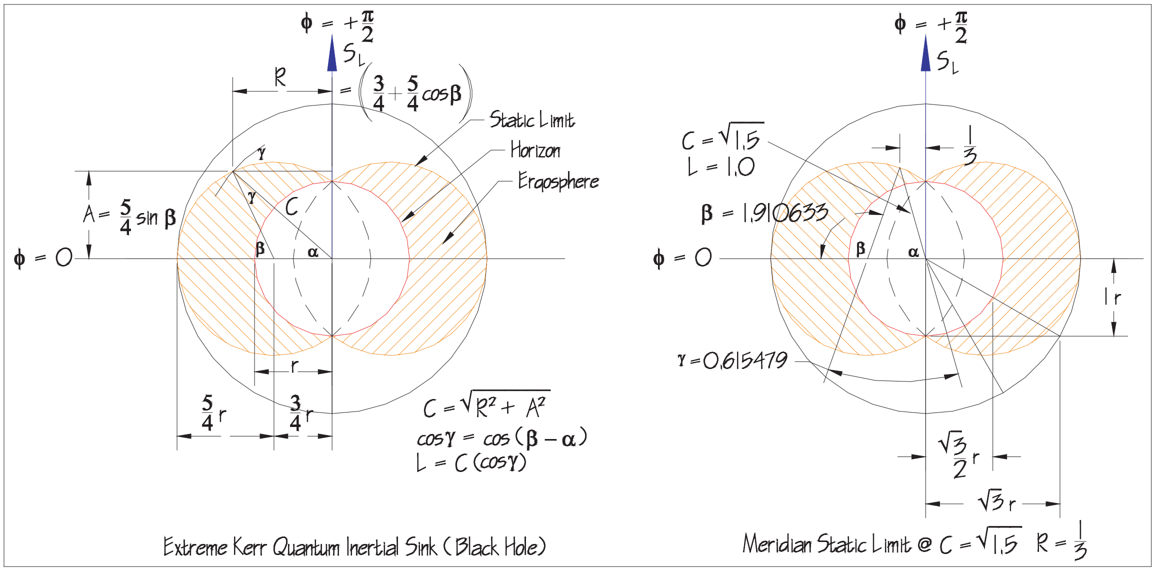
$$\pm\{a\} \equiv \frac{\cos(\omega_0 t - \theta)}{\left|\cos(\omega_0 t - \theta)\right|} a, \quad \mp\{a\} \equiv -\frac{\cos(\omega_0 t - \theta)}{\left|\cos(\omega_0 t - \theta)\right|} a \tag{5.11}$$

Obviously, θ and ϕ rotate at the same frequency, with the axis of the ϕ rotation rotating in the equatorial plane. This motion avoids the orientation entanglement condition and is necessitated by the assumed continuity condition of a classical spacetime continuum and the density property postulated in this development. When analyzed it is apparent that the motion is that of a transverse wave traveling in tight orbit around the spin axis, its amplitudes inclined toward the poles, analogous to a gravitationally bound, electromagnetic wave, and in fact constitutes the magnetic field of the quantum.

This diagram is a cross-section through the spin axis and shows the relationship of the static limit, the ergosphere, and the horizon. The ergosphere is the domain of the strong interaction. The transverse or ϕ differential is limited in its motion toward the spin poles to the point on the static limit where L = 1.

The metric simplifies at the horizon with no radial motion as

$$d\tau^2 = -dt^2 + 4r_{0n}dtd\theta - R^2 d\theta^2 \mp \left\{\cos^2(\omega_0 t - \theta) L^2 d\phi^2\right\} \tag{5.12}$$



Quantum Inertial Sink 1
Figure 14

From this diagram we have the following coefficient component for ϕ along the meridians at the static limit

$$L = \tfrac{4}{5} r_{0n} + \tfrac{3}{5} R = \tfrac{4}{5} r_{0n} + \tfrac{3}{5}\left(\tfrac{3}{4} + \tfrac{5}{4}\cos\beta\right) r_{0n} = \left(\tfrac{5}{4} + \tfrac{3}{4}\cos\beta\right) r_{0n} \qquad (5.13)$$

Substituting this in equation (5.12) simplifies at the horizon along the equatorial plane of a fixed spin axis where $\cos\beta = 1$, as

$$d\tau^2 = \left(idt - i2r_{0n}d\theta\right)^2 \mp \left\{\cos^2\left(\omega_0 t - \theta\right)\left(2r_{0n}\right)^2 d\phi^2\right\} \qquad (5.14)$$

The corresponding spacelike metrics is

$$d\sigma^2 = -\left(idt - i2r_{0n}d\theta\right)^2 \pm \left\{\cos^2\left(\omega_0 t - \theta\right)\left(2r_{0n}\right)^2 d\phi^2\right\} \qquad (5.15)$$

giving the fundamental symmetry

$$d\sigma^2 \equiv -d\tau^2 \qquad (5.16)$$

and for the proper time and space, indicating the orthogonal nature of space and time,

$$d\sigma \equiv i d\tau\,. \qquad (5.17)$$

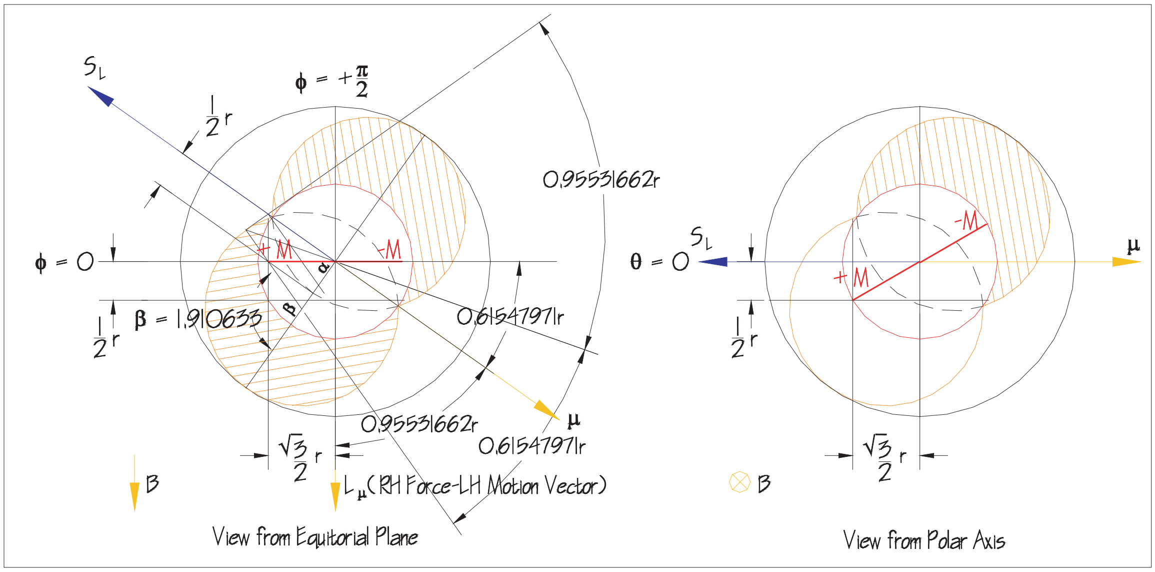
This can be represented by the following anti-symmetric orthonormal matrix at r_0,

		Direction of ortho normal vector dx_i with respect to		
		X Axis	Y Axis	Z Axis
Vector dx_i originating at	X = +1	0	$+rd\theta$	$+r\sin\omega td\phi$
	Y = +1	$-rd\theta$	0	$-r\cos\omega td\phi$
	Z = +1	$-r\sin\omega td\phi$	$+r\cos\omega td\phi$	0
	X = -1	0	$-rd\theta$	$-r\sin\omega td\phi$
	Y = -1	$+rd\theta$	0	$+r\cos\omega td\phi$
	Z = -1	$+r\sin\omega td\phi$	$-r\cos\omega td\phi$	0

Table 3 - Quantum Anti-Symmetric Orthonormal Matrix at r_0

In the presence of an anti-parallel external magnetic field as shown in Quantum Inertial Spin Diagram 2, the quantum spin axis inclines toward the equatorial plane and precesses about its initial position. The resulting coefficients of ½ spin can be seen here. Note also that the Heisenberg "observational" uncertainty is limited by the inverse curvature of the horizon to

$$r_0^2 c = m_{l0} r_0 c = \text{ת} c = \hbar \,. \tag{5.18}$$



Quantum Inertial Sink 2
Figure 15

Full Model Development

This model is elsewhere more fully developed and presented as the 3-D representation of a classical 4-D oscillation. Expansion acts as an EMF that drives the fundamental frequency, both by mechanical analogy and as the actual mechanical or piezoelectric basis for electro-magnetism. The rest-mass quantum is thus a small simple harmonic oscillator, with a potential-kinetic, capacitive-inductive energy cycle, in a general inductive mode during expansion, of which the waveform of ordinary matter is the result. During universal contraction, a capacitive mode ensues, resulting in a predominance of anti-matter.

Over a short period of time, in particular in the absence of confinement at nuclear density, expansion leads to a drop in mechanical impedance, resulting in a transmission of energy and power at the boundary of a neutral or resonant quantum. The result is beta decay, which is tuned to the expansion rate for any isolated neutral quantum and generates the electromagnetic interaction, which is properly considered an intra-action of the spacetime continuum. The rest-mass ratios between the neutron, electron and proton and the "missing" mass of beta decay arise naturally in this analysis. Finally, quark phenomenology of fractional charge is shown to be the property of the nodes and antinodes of the quantum waveform.

Endnote

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i A derivative taken on a flat rectilinear area,

$$\frac{\pm d\mathrm{A}}{dr}=\frac{(\mathrm{A}\pm d\mathrm{A})-\mathrm{A}}{dr}=\frac{(r\pm dr)^2-r^2}{dr}=\frac{\pm 2rdr+dr^2}{dr}$$

gives a differential area of

$$\pm d\mathrm{A}=\pm 2rdr+dr^2\,.$$

Now consider a hyperbolic surface, specifically the derivative of the inverse curvature of a pseudosphere, which is of constant negative curvature, for simplicity $k=-1$, where r_i is the interior radius and r_e is the exterior radius, and we have the function, where either r_e or r_i, could be used as the variable

$$k_{r_e}^{-1}=k^{-1}(r_e)=-r_e^{-1}r_e=-r_ir_e$$

The curvature is conserved, therefore the differential is zero or

$$dk_{r_e}^{-1}=(-r_i-dr_i)(r_e-dr_e)-(-r_ir_e)=r_idr_e-r_edr_i+dr_idr_e=0$$

The senses of the radii and their differentials indicate a direction toward (+) or away from (-) the exterior of the pseudosphere. Note that the differentials are of the same sense. Thus the above equation indicates a change toward the mouth or rim of the pseudosphere, as r_i is increasing and r_e is decreasing. At the point of normalization, where $r_i=r_e=r_0=1$,

we have

$$dr_e-dr_i=-dr_idr_e\,.$$

Therefore $dr_i=-x^{-1}, dr_e=-x \therefore dr_idr_e=1$ and after a sense inversion we have the solution

$$x-x^{-1}=1$$

$$x=\sqrt{\tfrac{5}{4}}+\tfrac{1}{2}=1.618033\ldots=\Phi$$

the well known coefficient of conservative evolution of a system.

Note that the product $(x)(x^{-1})1=1$ is conserved.

At the point where $r_i=\Phi^{-1}$ and $r_e=\Phi$,

we have, where the differential senses are explicit,

$$-r_i(-dr_e)+r_e(-dr_i)=-(-dr_i)(-dr_e)$$

and we can normalize the differentials at $k_{r_e}^{-1}(\Phi)$ as

$$|dr_i|=|dr_e|=|dr_o|$$

giving

$$\frac{dr_i}{dr_e}=\frac{dr_e}{dr_i}=1\ ,$$

therefore

$$dr_i^2=dr_e^2=dr_0^2=d\mathrm{A}_0\,.$$

Then the invariant inverse curvature is equal to the square of normalized differentials

$$k_{r_e}^{-1}(\Phi)=-r_ir_e=-dr_idr_e=-dr_0^2=-d\mathrm{A}_0=-1.$$

However, for <u>any</u> such conservative hyperbolic system of any invariant finite curvature, we can state the following,

$$dr_i=r_i\Phi^{-1},\ dr_e=r_e\Phi\ ,$$

so that

$$dr_i dr_e = \left| r_i r_e \right|$$

and we have the following relationship between the inverse curvature function and its differential components

$$k_{r_e}^{-1} = k_{r_e}^{-1} + dk_{r_e}^{-1} = r_i dr_e - r_e dr_i = -r_i r_e \Phi + r_e r_i \Phi^{-1} = -dr_i dr_e = -r_i r_e \, .$$

Finally, with some substitution, for the function and its derivative, as

$$dr_i = \frac{r_i}{\Phi^2 r_e} dr_e \text{ and } r_i = \frac{-k_{r_e}^{-1}}{r_e}$$

we have, with rearrangement and simplification

$$k_{r_e}^{-1} + dk_{r_e}^{-1} = \frac{-k_{r_e}^{-1}}{r_e} dr_e - \frac{-k_{r_e}^{-1}}{\Phi^2 r_e} dr_e = -\left(1 - \Phi^{-2}\right) k_{r_e}^{-1} d\,\boldsymbol{ln}\, r_e = -\Phi^{-1} k_{r_e}^{-1} d\,\boldsymbol{ln}\, r_e \, .$$

The symmetrical condition for $k_{r_i}^{-1}$ is

$$k_{r_i}^{-1} + dk_{r_i}^{-1} = \frac{-k_{r_i}^{-1}\Phi^2}{r_i} dr_i - \frac{-k_{r_i}^{-1}}{r_i} dr_i = -\left(\Phi^2 - 1\right) k_{r_i}^{-1} d\,\boldsymbol{ln}\, r_i = -\Phi k_{r_i}^{-1} d\,\boldsymbol{ln}\, r_i \, ,$$

and obviously

$$d\,\boldsymbol{ln}\, r_i = \Phi^{-1}\boldsymbol{,}\; d\,\boldsymbol{ln}\, r_e = \Phi \, .$$

Since

$$-dr_i dr_e = -\Phi^{-1} k_{r_e}^{-1} d\,\boldsymbol{ln}\, r_e = -\Phi k_{r_i}^{-1} d\,\boldsymbol{ln}\, r_i$$

we have

$$dk_{r_i r_e}^{-1} = \Phi^{-1} k_{r_e}^{-1} d\,\boldsymbol{ln}\, r_e - \Phi k_{r_i}^{-1} d\,\boldsymbol{ln}\, r_i = 0 \, ,$$

and finally

$$k^{-1} = d\mathrm{A}_0 = -dr_0^2 = -dr_i dr_e = -\Phi^{-1} k_{r_e}^{-1} d\,\boldsymbol{ln}\, r_e = -\Phi k_{r_i}^{-1} d\,\boldsymbol{ln}\, r_i \, .$$

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